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Abstract

A similarity solution of a binary chemical reaction on MHD unsteady non-Newtonian Casson fluid flow with heat and mass transfer past a flat porous plate is investigated. The effects of Casson parameter on the velocity profiles for both cooling and heating problems are studied and we also study the effects of exothermic/endothermic chemical reaction rate and Arrhenius activation energy on the temperature and concentration respectively in this paper. The governing partial differential equations are reduced to ordinary differential equations by introducing locally similarity transformation [Maleque (2010)]. Numerical solutions to the reduced non-linear similarity equations are then obtained by adopting six orderd implicit Runge-Kutta and shooting method using the Nachtsheim- Swigert iteration technique. The results of the numerical solution are then presented graphically in the form of velocity, temperature and concentration profiles. The corresponding skin friction coefficient, the Nusselt number and the Sherwood number are also calculated and displayed in table showing the effects of various parameters on them.

Keywords: MHD Non-Newtonian Casson fluid; Exothermic/Endothermic Chemical Reaction; Activation Energy; Porous Plate

1. Introduction

In free convection boundary layer flows with simultaneous heat mass transfer, one important criteria that is generally not encountered is the species chemical reactions with finite Arrhenius activation energy. The Arrhenius law is usually of the form [Michal etal (2004)]

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where $K$ is the rate constant of chemical reaction and $B$ is the pre-exponential factor simply prefactor (constant), is based on the fact that increasing the temperature frequently causes a marked increase in the rate of reactions. $E_a$ is the activation energy and $k = 8.61 \times 10^{-5} \text{ eV} / \theta K$ is the Boltzmann constant which is the physical constant relating energy at the individual particle level with temperature observed at the collective or bulk level. It is the gas constant $R$ divided by the Avogadro constant $N_A$

$$k = \frac{R}{N_A}. \quad (2)$$

In areas such as geothermal or oil reservoir engineering, the above phenomenon is usually applicable. Apart from experimental works in these areas, it is also important to make some theoretical efforts to predict the effects of the activation energy in flows mentioned above. But in this regard very few theoretical works are available in literature. The reason is that the chemical reaction processes involved in the system are quite complex and generally the mass transfer equation that is required for all the reactions involved also become complex. Theoretically, such an equation is rather impossible to tackle. Form chemical kinetic viewpoint this is a very difficult problem, but if the reaction is restricted to binary type a lot of progress can be made. The thermo-mechanical balance equations for a mixture of general materials were first formulated by Truesdell (1957). Thereafter Mills, N (1966) and Beevers et al. (1982) have obtained some exact solutions for the boundary layer flow of a binary mixture of incompressible Newtonian fluids. Several problems relating to the mechanics of oil and water emulsions, particularly with regard to applications in lubrication practice, have been considered within the context of a binary mixture theory by Al-Sharif et. al. (1993) and Wang et. al. (1993).

A simple model involving binary reaction was studied by Bestman (1990). He considered the motion through the plate to be large which enabled him to obtain analytical solutions (subject to same restrictions) for various values of activation energy by employing the perturbation technique proposed by Singh and Diskshit (1988). Bestman (1991) and Alabraba et al. (1992) took into account the effect of the Arrhenius activation energy under the different physical conditions.

Kandasamy et. al. (2005) studied the combined effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection. Their result shows that the flow field is influenced appreciably by chemical reaction, heat source and suction or injection at the wall of the wedge. More recently Makinde et. al. (2011a, 2011b) studied the problems of unsteady convection with chemical reaction and radiative heat transfer past a flat porous plate moving through a binary mixture in an optically thin environment. Recently Maleque (2013a, 2013), investigated a numerical solution of unsteady natural convection heat and mass transfer boundary layer flow past a flat porous plate taking into account the effect of Arrhenius activation energy. Maleque (2013b) studied heat and mass transfer boundary layer flow with a binary exothermic chemical reaction and the effects of exothermic/ endothermic chemical reaction in presence of thermal radiation investigated by Maleque (2013c). More recently Maleque (2013d, 2014) investigated the numerical solution of unsteady natural convection heat and mass transfer boundary layer flow past a vertical porous plate taking into account the effects of viscous
dissipation, heat generation/absorption, and chemical reaction with Arrhenius activation energy in the presence of uniform magnetic field.

Fig. 1. The flow configuration and the coordinate system.

Above all works have been studied for Newtonian fluid flow. In many coating applications in the polymer processing industry especially in extraction of crude oil from petroleum products however fluids used to generate damage resistant surfacing are generally non-Newtonian. Several studies have been published concerning non-Newtonian flow by Acrivos et al (1960), Mitschka (1965), Rogovskii and Gorbis (1975), Sarma and Gupta (1981), Ma and Hwang (1990) and Andersson et al (2001). They presented a similarity solution for the power low fluid flow over a rotating disk. Rashaida et al (2006) considered both flow and species from a rotating dish to a Bingham plactic. Recently Beg and Maleque et al (2012) examined analytically the steady hydrodynamic flow of a power-low non-Newtonian fluid from a rotating disk to a saturated non-Darcian porous medium.

In the category of non-Newtonian fluids, Casson fluid has distinct features. This model was presented by Casson (1959) for the flow of viscoelastic fluid in 1959. This model is cast off by fuel engineers in the description of adhesive slurry and is improved for forecasting high shear-rate viscosities when only low and transitional shear-rate data are accessible. Examples of Casson fluid include jelly, tomato sauce, honey, soup and concentrated fruit juices, etc. Human blood can also be treated as Casson fluid. Excellent studies of non Newtonian Casson fluid flow have been published by several authors. Eldabe et al (1995) studied the heat transfer of MHD non-Newtonian Casson fluid flow between two rotating cylinders and Dash et. al (1996) investigated Casson fluid flow in a pipe filled with a homogeneous porous medium. Analysis of the Casson non-Newtonian blood models in steady and oscillatory flow. Mostafa et al (2011) investigated the unsteady boundary layer flow of a Casson fluid due to an impulsively started moving flat plate and Nadeem et al (2012) studied the Casson MHD fluid over an exponentially shrinking sheet. Recently Mukhopadhyay et al (2013) studied the unsteady two dimensional flow of a non-Newtonian Casson fluid over a stretching surface. Very recently Nadeem et al (2013) considered magnetohydrodynamic Casson fluid flow in two lateral directions past a porous linear stretching sheet. In the present paper we investigate an exothermic/endothermic binary chemical reaction on unsteady MHD non-Newtonian casson fluid flow with heat and mass transfer. The effect of the Casson parameter on the velocity profiles for cooling and heating plate and the effects of exothermic/endothermic
chemical reaction rate and Arrhenius activation energy on the concentration are also studied in this paper. This problem is an extension work studied by Maleque (2014).

2. Governing Equations

Let us consider an unsteady free convective heat and mass transfer MHD laminar boundary layer flow of an electrically conducting viscous fluid past an infinite vertical porous plate \( y = 0 \). The flow is assumed to be moving in the upward direction with a constant velocity \( U_0 \) and the exothermic and endothermic chemical reactions are considered. We take \( x \)-axis along the plate in upward direction and \( y \)-axis is normal to it. A uniform magnetic field \( \vec{B} \) parallel to \( y \)-axis is applied to the plate which is electrically non-conducting. The magnetic field is of the form \( \vec{B} = (0, B_0, 0) \). The equation of conservation of electric charge is \( \nabla \cdot \vec{J} = 0 \), where \( \vec{J} = (J_x, J_y, J_z) \), the direction of propagation is considered only along the \( y \)-axis and does not have any variation along the \( y \)-axis.

The derivative of \( J_y \) with respect to \( y \) is namely \( \frac{\partial J_y}{\partial y} = 0 \), which gives rise to \( J_y = \text{constant} \). Since the plate is electrically non-conducting, this constant is zero and hence \( J_y = 0 \) everywhere in the flow. It is assumed that the plate is infinite in extent and hence all the physical quantities depend on \( y \) and \( t \).

The physical model and geometrical coordinates are shown in above Fig-1. MHD incompressible laminar flow of a homogeneous fluid satisfies the following continuity, momentum, energy and mass equations:

\[
\frac{\partial v}{\partial y} = 0
\]

\[
\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \nu \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} + \frac{g \beta T}{\rho} (T - T_\infty) + \frac{g \beta^*}{\rho} (C - C_\infty) - \frac{\sigma B_0^2}{\rho} u,
\]

\[
\rho c_p \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} \right) = \kappa \frac{\partial^2 T}{\partial y^2} + \gamma k_r^2 (T - T_\infty)^w \exp \left[ -\frac{E_a}{k(T-T_\infty)} \right] (C - C_\infty),
\]

\[
\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D_m \left( \frac{\partial^2 C}{\partial y^2} - k_r^2 (T - T_\infty)^w \exp \left[ -\frac{E_a}{k(T-T_\infty)} \right] (C - C_\infty),
\]

The boundary conditions of above system are

\[
\begin{cases}
  u = U_0, v = v_w(t), T = T_w, C = C_w \text{ at } y = 0, \quad t > 0, \\
  u = 0, T \to T_w, C \to C_w \text{ at } y = \infty, \quad t > 0.
\end{cases}
\]

Where \( (u, v) \) is the velocity vector, the plate is moving with a uniform velocity \( U_0 \), Casson parameter \( \beta \), \( T \) is the temperature, \( C \) is the concentration of the fluid, \( \nu \) is the kinematic coefficient of viscosity, \( \kappa \) is the heat diffusivity coefficient, \( \beta_T \) and \( \beta^* \) are the coefficients of volume expansions for temperature and concentration respectively, \( D_m \) is the coefficient of mass...
diffusivity, \( k_r^2 \) is the chemical reaction rate constant, \( \gamma(\pm 1) \) is exothermic/endothermic parameter {Maleque (2013a)} and \((T - T_\infty)^w \exp[-E_a/k(T - T_\infty)]\) is the Arrhenius function where \( w \) is a dimensionless constant exponent fitted rate constants typically lie in the range \(-1 < w < 1\). [Maleque (2013a)].

### 3. Mathematical Formulations

In order to solve the governing equations (3) to (6) under the boundary conditions (7), we adopt the well defined similarity technique to obtain the similarity solutions. For this purpose the following non-dimensional variables are now introduced:

\[
\eta = \frac{y}{\delta(t)}, \quad \frac{u}{U_0} = f(\eta), \quad \frac{T - T_\infty}{T_w - T_\infty} = \theta(\eta) \quad \text{and} \quad \frac{C - C_\infty}{C_w - C_\infty} = \phi(\eta)
\]

From the equation of continuity (3) we have

\[
v(t) = -\frac{\nu_0 U}{\delta(t)}
\]

where \( \nu_0 = -\nu_0 \delta / \nu \) is the dimensionless suction / injection velocity at the plate, \( \nu_0 > 0 \) corresponds to suction and \( \nu_0 < 0 \) corresponds to injection.

Introducing the dimensionless quantities from equation (8) and \( v \) from equation (9) in equations (4), (5) and (6), we finally obtain the nonlinear ordinary differential equations as

\[
\begin{align*}
1 + \frac{1}{\beta} f'' + \eta \frac{\delta \theta'}{\nu} f' + v_0 f' + G_r \theta + G_m \phi - M f &= 0 \\
Pr^{-1} \theta'' + \left( \eta \frac{\delta \theta'}{\nu} + v_0 \right) \theta' + \gamma \lambda^2 e^{-\frac{E}{\theta}} \theta^w \phi &= 0 \\
Sc^{-1} \phi'' + \left( \eta \frac{\delta \phi'}{\nu} + v_0 \right) \phi' - \lambda^2 \frac{E}{\theta^w} \theta^w \phi &= 0
\end{align*}
\]

Here, Grashof number \( G_r = g \beta (T_w - T_\infty) \delta^2 / \nu U_0 \), Modified (Solutal) Grashof number \( G_m = g \beta (C_w - C_\infty) \delta^2 / \nu U_0 \), Prandtl Number \( Pr = \rho \nu c_p / \kappa \), Magnetic parameter \( M = \sigma \delta^2 B_0^2 / \mu \), Chemical reaction rate constant \( \lambda^2 = k_r^2 \delta^2 (T_w - T_\infty)^w / \nu \), Schmidt number \( Sc = \nu / D_M \), The non-dimensional activation energy \( E = E_a / k(T_w - T_\infty) \).

The equations (10) to (12) are similar except for the term \( \delta \theta' / \nu \), where time \( t \) appears explicitly. Thus the similarity condition requires that \( \delta \theta' / \nu \) must be a constant quantity. Hence following Maleque [35] one can try a class of solutions of the equations (10) to (12) by assuming that

\[
\frac{\delta \theta'}{\nu} = A \quad \text{(Constant)}
\]

From equation (13) we have
\[ \delta(t) = \sqrt{\frac{2A}{\nu t}} + L \]  

(14)

where the constant of integration \( L \) is determined through the condition that \( \delta = L \) when \( t = 0 \). Here \( A = 0 \) implies that \( \delta = L \) represents the length scale for steady flow and \( A \neq 0 \) that is, \( \delta \) represents the length scale for unsteady flow. Let us now consider a class of solutions for which \( A = 2 \) (Maleque[14]) and hence the length scale \( \delta \) from equation (14) becomes

\[ \delta(t) = 2\sqrt{\frac{A}{\nu t}} + L \]  

(15)

which exactly corresponds to the usual scaling factor considered for various unsteady boundary layer flow (Schlichting [36], Maleque [35]). Since \( \delta \) is a scaling factor as well as similarity parameter any other values of \( A \) in equation (14) would not change the nature of the solution except that the scale would be different. Finally introducing equation (13) in equations (10) to (12) respectively, we have the following dimensionless non-linear ordinary differential equations

\[
\left(1 + \frac{1}{\beta}\right)f'' + (2\eta + \nu_0) f' + G_r \theta + G_m \phi - M f = 0
\]

(16)

\[
P_r^{-1} \theta'' + (2\eta + \nu_0) \theta' + \gamma \lambda^2 e^{-\frac{E}{\theta}} \theta^w \phi = 0
\]

(17)

\[
S_c^{-1} \phi'' + (2\eta + \nu_0) \phi' - \lambda^2 e^{-\frac{E}{\phi}} \theta^w \phi = 0
\]

(18)

The boundary conditions equation (7) then becomes

\[
\begin{align*}
f(0) &= 1, \theta(0) = 1, \phi(0) = 1, \\
f(\infty) &= 0, \theta(\infty) = 0, \phi(\infty) = 0.
\end{align*}
\]

(19)

In all over equations primes denote the differentiation with respect to \( \eta \). Equations (16) to (18) are solved numerically under the boundary conditions (19) using Nachtsheim-Swigert iteration technique.

**4. Numerical Solutions**

Equations (16) –(18) are solved numerically under the boundary conditions (19) using Nachtsheim-Swigert (1965) iteration technique. In equation (19) there are three asymptotic boundary conditions and hence follows three unknown surface conditions \( f'(0), \theta'(0) \) and \( \phi'(0) \). Within the context of the initial-value method and the Nachtsheim-Swigert iteration technique the outer boundary conditions may be functionally represented by the first order Taylor's series as

\[
\begin{align*}
f(\eta_{\text{max}}) &= f(x, y, z) = f_c(\eta_{\text{max}}) + \Delta X f_X + \Delta Y f_Y + \Delta Z f_Z = \delta_1, \\
\theta(\eta_{\text{max}}) &= \theta(x, y, z) = \theta_c(\eta_{\text{max}}) + \Delta X \theta_X + \Delta Y \theta_Y + \Delta Z \theta_Z = \delta_2, \\
\phi(\eta_{\text{max}}) &= \phi(x, y, z) = \phi_c(\eta_{\text{max}}) + \Delta X \phi_X + \Delta Y \phi_Y + \Delta Z \phi_Z = \delta_3,
\end{align*}
\]

(20)

(21)

(22)

with the asymptotic convergence criteria given by

\[
\begin{align*}
f'(\eta_{\text{max}}) &= f'(x, y, z) = f'_c(\eta_{\text{max}}) + \Delta X f'_X + \Delta Y f'_Y + \Delta Z f'_Z = \delta_4, \\
\theta'(\eta_{\text{max}}) &= \theta'(x, y, z) = \theta'_c(\eta_{\text{max}}) + \Delta X \theta'_X + \Delta Y \theta'_Y + \Delta Z \theta'_Z = \delta_5,
\end{align*}
\]

(23)

(24)
\[
\phi'(\eta_{\text{max}}) = \phi'(X,Y,Z) = \phi_c'(\eta_{\text{max}}) + \Delta X \phi'_X + \Delta Y \phi'_Y + \Delta Z \phi'_Z = \delta_6, \tag{25}
\]
where \( X = f'(0), \ Y = \theta'(0), \ Z = \phi'(0) \) and \( X, Y, Z \) subscripts indicate partial differentiation, e.g., \( f'_Y = \frac{\partial f'(\eta_{\text{max}})}{\partial \theta'(0)} \). The subscript \( c \) indicates the value of the function at \( \eta_{\text{max}} \) to be determined from the trial integration.

Solutions of these equations in a least square sense requires determining the minimum value of \( E = \delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2 + \delta_5^2 + \delta_6^2 \) with respect to \( X, Y \) and \( Z \). To solve \( \Delta X, \Delta Y \) and \( \Delta Z \) we require to differentiate \( E \) with respect to \( X, Y \) and \( Z \) respectively. Thus adopting this numerical technique, a computer program was set up for the solutions of the basic non-linear differential equations of our problem where the integration technique was adopted as the six ordered Range-Kutta method of integration. The results of this integration are then displayed graphically in the form of velocity, temperature and concentration profiles in Figs. (1)-(9). In the process of integration, the local skin-friction coefficient, the local rates of heat and mass transfer to the surface, which are of chief physical interest are also calculated out. The equation defining the wall skin-friction is

\[
\tau = -\mu \left(1 + \frac{1}{\beta} \left( \frac{\partial u}{\partial y} \right) \right)_{y = 0} = -\mu \frac{U_0}{\delta} \left(1 + \frac{1}{\beta} \right) f'(0)
\]

Hence the skin-friction coefficient is given by

\[
\frac{1}{2} C_f = \frac{\tau}{\rho U_0^2} \Rightarrow \frac{1}{2} Re C_f = \left(1 + \frac{1}{\beta} \right) f'(0), \tag{26}
\]

here, the Reynolds number \( Re = \rho U_0 \delta / \mu \).

The heat flux \( (q_w) \) and the mass flux \( (M_w) \) at the wall are given by

\[
q_w = -\kappa \left( \frac{\partial T}{\partial y} \right)_{y = 0} = -\kappa \frac{\Delta T}{\delta} \theta'(0) \quad \text{and} \quad M_w = -D_M \left( \frac{\partial C}{\partial y} \right)_{y = 0} = -D_M \frac{\Delta C}{\delta} \phi'(0).
\]

Hence the Nusselt number \( (N_u) \) and the Sherwood number \( (S_h) \) are obtained as

\[
N_u = \frac{q_w \delta}{\kappa \Delta T} \alpha - \theta'(0), \tag{27}
\]

\[
\text{and} \quad S_h = \frac{M_w \delta}{D_M \Delta C} \alpha - \phi'(0). \tag{28}
\]

These above coefficients are then obtained from the procedure of the numerical computations and are sorted in Table 1.
5. Results and Discussions

The parameters entering into the fluid flow are Grashof number $G_r$, Solutal (modified) Grashof number $G_m$, suction parameter $v_0$, Prandtl number $P_r$, Schmidt number $S_c$, Casson parameter $\beta$, the non-dimensional chemical reaction rate constant $\lambda^2$, and the non-dimensional activation energy $E$. It is, therefore, pertinent to inquire the effects of variation of each of them when the others are kept constant. The numerical results are thus presented in the form of velocity profiles, temperature profiles and concentration profiles in Figs. 2-10 for the different values of $\beta, G_r, G_m, \lambda, E$ and $v_0$. The value of $G_r$ is taken to be both positive and negative, since these values represent respectively cooling and heating of the plate. The values of $G_r$ is taken to be large (
Since the value corresponds to a cooling problem that is generally encountered in nuclear engineering in connection with the cooling of reactors. In air ($P_r = 0.71$) the diluting chemical species of most common interest have Schmidt number in the range from 0.6 to .75. Therefore, the Schmidt number $S_c = 0.6$ is considered. In particulars, 0.6 corresponds to water vapor that represents a diffusing chemical species of most common interest in air. The values of the suction parameter $\nu_0$ are taken to be large. Apart from the following Figures and Tables, the representative velocity, temperature and concentration profiles and the values of the physically important parameters, i.e. the local shear stress, the local rates of heat and mass transfers, are illustrated for uniform wall temperature and species concentration in Figures 2—14 and table-1.

**Fig. 4.** Effect of $G_r$ on the velocity

**Fig. 5.** Effects of $\gamma$ and $E$ on the velocity

5.1 Effects of casson parameter $\beta$ and Grashof number $G_r$ on the velocity profiles for Cooled and heated plate

The effect of Casson parameter $\beta$ on the velocity profiles for mixed convective flow ($G_r = 1$) shown in Fig.2. The figure shows the velocity decreases with the increasing values of Casson parameter $\beta$. That is the effect of increasing values of $\beta$ is to reduce the boundary layer thickness. For cooled plate ($G_r = -20$), it has been observed that the velocity profiles decrease with the increasing values of Casson parameter $\beta$ shown in Fig.3. From this figure it has been also investigated that the velocity profiles increase in the boundary layer $0 < \eta < 0.4$ but decrease for
\( \eta > 0.4 \) with the increasing values of \( \beta \) for heated plate \((G_r = 20)\). That is boundary layer separation is found for heated plate. The velocity profiles generated due to impulsive motion of the plate is plotted in Fig.4 for both heated \((G_r > 0)\) and cooled \((G_r < 0)\) of the plate keeping other parameters fixed \((G_m = 1.0, \beta = 1.0, Pr = 0.71, S_c = 0.6, \lambda = 5, v_0 = 3.0, E = 1.0 \text{ and } w = 1)\). In Fig.4, velocity profiles are shown for different values of \( G_r \). We observe that velocity increases with increasing values of \( G_r \) for the heated plate. From this figure it is also observed that the negative increase in the Grashof number leads to the decrease in the velocity field. That is, for cooled plate (Fig.4), the effects of the Grashof number \( G_r \) on the velocity field have also opposite effects, as compared to the heated plate. When \( G_r = 1 \), that is, when the temperature gradient is small the velocity profile shows its usual trend of gradual decay.

5.2. Effects of Activation Energy \((E)\) on the concentration, the temperature and the velocity profiles for exothermic/endothermic chemical reaction

In chemistry, **Activation Energy** is defined as the energy that must be overcome in order for a chemical reaction to occur. Activation energy may also be defined as the minimum energy required starting a chemical reaction. The activation energy of a reaction is usually denoted by \( E_a \) and given in units of kilojoules per mole. An exothermic reaction is a chemical reaction that releases energy in the form of light and heat. It is the opposite of an endothermic reaction. It gives out energy to its surroundings. The energy needed for the reaction to occur is less or greater than the total energy.
released for exothermic or endothermic reaction respectively. Energy is obtained from chemical bonds. When bonds are broken, energy is required. When bonds are formed, energy is released. Each type of bond has specific bond energy. It can be predicted whether a chemical reaction will release or require heat by using bond energies. When there is more energy used to form the bonds than to break the bonds, heat is given off. This is known as an exothermic reaction. When a reaction requires an input or output of energy, it is known as an exothermic or endothermic reaction.

Effects of Activation Energy (E) and exothermic parameter (γ) on the temperature, the concentration and velocity profiles are shown in fig.4 to fig.6 respectively. γ = 1 and γ = −1 represent exothermic and endothermic chemical reactions respectively. The parameter E does not enter directly into the momentum equation but its influence comes through the mass and energy equations. Fig.4 shows the variation of the velocity profiles for different values of E. From this figure it has been observed that the velocity profile mark increases with the increasing values of E for endothermic chemical reaction. But negligible effects are found for exothermic reaction. Fig.6 shows that a small decreasing effect of temperature profile is found for increasing values of E for exothermic reaction (γ = 1) but mark opposite effects are found for endothermic reaction in Fig.6. That is the temperature profile increases for increasing values of E for endothermic reaction γ = −1. From equation (1) we observe that chemical reaction rate (K) decreases with the increasing values of activation energy (Ea). We also observe from equation (20) that increase in activation energy (E) leads to decrease λ\exp(-E/\theta) as well as to increase the concentration profiles shown in Fig.7. Small and reported effects are found for γ = 1 and γ = −1 respectively.

Fig. 8. Effects of γ and λ on the velocity profiles.

Fig. 9. Effects of γ and λ on the Temperature profiles.
5.3. Effects of $\lambda$ on the concentration, the temperature and the velocity profiles for exothermic/endothermic reaction

Considering chemical reaction rate constant $\lambda^2$ is always positive. Figures 7-9 represent the effect of chemical rate constant $\lambda$ on the velocity, the temperature and the concentration profiles respectively.

We observe from the figure 7 and figure 8 that velocity and temperature profiles increase with the increasing values of $\lambda$ for exothermic reaction but opposite effects are found in these figures for endothermic reaction. It is observed from the equation (1) that increasing temperature frequently causes a marked increase in the rate of reactions is shown in Fig.8. As the temperature of the system increases(Fig 9), the number of molecules that carry enough energy to react when they collide also increases. Therefore the rate of reaction increases with temperature. As a rule, the rate of a reaction doubles for every 10°C increase in the temperature of the system.

Last part of equation (20) that $\lambda^2 \exp(-E/\Theta)$ increases with the increasing values of $\lambda$. We also observe from this equation that increase in $\lambda^2 \exp(-E/\Theta)$ means increase in $\lambda$ leads to the decrease in the concentration profiles. This is in great agreement with Fig.10 for both cases $\gamma = \pm 1$. It is also observed from this figure that for exothermic reaction, the mass boundary layer is close to the plate other that endothermic reaction.

![Fig. 10. Effects of $\gamma$ and $\lambda$ on the concentration profiles](image1)

![Fig. 11. Effects of $M$ on the velocity](image2)
5.4 Effects of $M$ on the velocity profiles:
The effects of magnetic interaction parameter $M$ on velocity profiles is shown in Fig.11. Imposition of a magnetic field to an electrically conducting fluid creates a drag like force called Lorentz force. The force has the tendency to slow down the flow around the plate at the expense of increasing its temperature. This is depicted by decreases in velocity profiles as magnetic parameter $M$ increases as shown in Fig.11.

\[ v_0 = 3.0 \]
\[ v_0 = 0 \]
\[ v_0 = -3.0 \]

\[ \eta \]

\[ f \]

\[ \theta \]

**Fig. 12.** Effects of $v_0$ on the velocity profiles.

**Fig. 13.** Effects of $v_0$ on the temperature profiles.

5.5 Effects of suction/injection ($v_0$) on the velocity, the Temperature and the concentration profiles

The effects of suction and injection ($v_0$) for $\lambda = 5$, $E = 1$, $\beta = 50$, $\gamma = 1$, $Gr = 5$, $Gm = 1$, $Sc = 0.6$ and $Pr = 0.7$ on the velocity profiles, temperature profiles and concentration profiles are shown respectively in Fig.11 to Fig.13. For strong suction ($v_0 > 0$), the velocity, the temperature and the concentration profiles decay rapidly away from the surface. The fact that suction stabilizes the boundary layer is also apparent from these figures. As for the injection ($v_0 < 0$), from Fig.12 to Fig.14 it is observed that the profiles overshoot and close to the boundary. From Fig.12 it is observed that for strong suction the velocity is small while for injection the boundary layer is increasingly blown away from the surface of the plate to form an interlayer between the injection.
and the outer flow regions. Fig. 12 also depicts that for high values of injection parameter \((v_0 < 0)\), the velocity near the plate (for small values of \(\eta\)) is lower than that for smaller values of injection parameter \((v_0 < 0)\). Fig 13 shows the usual decay of temperature for strong suction while the thermal boundary layer increases rapidly for strong injection. It is seen from Fig. 14 that the species concentration within the boundary layer is higher for injection and lower for suction.

\[ \text{Fig. 14. Effects of } v_0 \text{ on the concentration profiles.} \]

\[ \text{Fig. 15. Comparison of our calculated velocity profile and the velocity profile of Bestman}\[7\] and the outer flow regions. \]

5.6 The skin-friction, the heat transfer and the mass transfer coefficients:

The skin-friction, the heat transfer and the mass transfer coefficients are tabulated in Table-1 for different values of \(Gr, \lambda, E, w\) and \(v_0\). We observe from Table-1 that the skin-friction coefficients decrease with increase in the Grashop number \(Gr\). It has been observed from Fig. 1 that the velocity profiles over shoot from the boundary layer for the values of Grashop number \(Gr = 20\) and \(Gr = 50\). It indicates that the gradients of the velocities \(f'(\eta)\) are positive for \(Gr = 20\) and \(Gr = 50\) at \(\eta = 0\). Therefore the skin friction coefficient \((-f'(\eta))\) for \(Gr = 20\) is negative shown in table-1. Again Fig. 1 also shows that the velocity profiles gradually decay to the surface and the boundary
layers are close to the plate for $Gr \leq 10$. It means that the gradient of the velocity $(f'(\eta))$ is negative at $\eta = 0$ for $Gr \leq 10$ indicate that the skin friction coefficient $(-f'(\eta))$ is positive shown in table-1. It is also observed from table-1 that the rate of mass transfer coefficient increases with the increasing values of $\lambda$ and the mass transfer coefficient decreases with the increasing values of $E$ and $w$. Finally it is concluded from this table that this is an increasing effect of suction parameter $v_0$ on the skin friction coefficient, heat and mass transfer coefficients.

5.7 Comparison between our numerical result and the analytical result of Bestman [7]

Bestman studied steady natural convective boundary layer flow with large suction. He solved his problem analytically by employing the perturbation technique proposed by Singh and Diskshit(1988). In our present work, taking $A = 0$ in equation (14) for considering steady flow. The values of the suction parameter $v_0^2 = 10$ is taken to see the effects of large suction. $Gr = Gm = 1.0$, $\beta = 50$, $\lambda = 5$, $E = 5$ and $w = 1$ are also chosen with a view to compare our numerical results with the analytical results of Bestman (1990). The comparison of velocity profiles as seen in Fig.15 highlight the validity of the numerical computations adapted in the present investigation.

In above all the computations the step size $\Delta \eta = 0.01$ was selected that satisfied a convergence criterion of $10^{-6}$ in almost all of different phases mentioned above. Stating $\eta_\infty = \eta_\infty + \Delta \eta$, the value of $\eta_\infty$ was found to each iteration loop. $(\eta_\infty)_{\text{max}}$, to each group of the parameters, has been obtained when value of unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than $10^{-6}$. However, different step sizes such as $\Delta \eta = 0.01$, $\Delta \eta = 0.005$ and $\Delta \eta = 0.001$ were also tried and the obtained solutions have been found to be independent of the step sizes observed in Fig.16.
Table 1 Effects of $Gr$, $\lambda$, $w$, $E$ and $v_0$ on the skin friction coefficient, Nusselt number and the Sherwood number.

<table>
<thead>
<tr>
<th>$Gr$</th>
<th>$-f'(0)$</th>
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<th>$-\phi'(0)$</th>
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<td>-10.0000</td>
<td>6.28618</td>
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<td>5.0000</td>
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<td>2.59852</td>
<td>2.83923</td>
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<tr>
<td>10.0000</td>
<td>0.24525</td>
<td>2.60349</td>
<td>2.83849</td>
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<tr>
<td>20.0000</td>
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<td>2.60609</td>
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<table>
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<tr>
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<td>0.30840</td>
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<td>0.0000</td>
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6. Conclusions

In this paper, we investigate the effects of chemical reaction rate and Arhenious activation energy on an unsteady natural convection heat and mass transfer boundary layer flow past a flat porous plate for non-Newtonian fluid. The Nachtsheim and Swigert [37] iteration technique based on sixth-order Range-Kutta and Shooting method has been employed to complete the integration of the resulting solutions.

The following conclusions can be drawn as a result of the computations:

The velocity decreases with the increasing values of Casson parameter $\beta$. That is the effect of increasing values of $\beta$ is to reduce the boundary layer thickness. Velocity increases for the cooling of the plate and the heating of the plate leads to the decrease in the velocity field. That is, for heating of the plate the effects of the Grashof number $G_r$ on the velocity field have also opposite effects, as compared to the cooling of the plate. As Grashof number becomes larger the velocity profile overshoot the uniform velocity close to the boundary. The temperature profiles increase for increasing values of $E_a$. Increase in $\lambda$ leads to the decrease in the concentration profiles. The velocity profile decreases with the increasing values of $\lambda$. Increase in activation energy (E) leads to the increase of the concentration, For small changes of Activation energy that must occur to convert the reactants into the product of the reaction. Energy must be invested to begin breaking the reactant bond so that product bond can form. For strong suction ($v_0 > 0$), the velocity, the temperature and the concentration profiles decay rapidly away from the surface. As for the injection ($v_0 < 0$), it is observed that the boundary layer is increasingly blown away from the plate to form an interlayer between the injection and the outer flow regions. Finally Activation energy and chemical reaction have mark effects on velocity, temperature and concentration profiles.

7. Nomenclature

- $(u, v)$: components of velocity field (m/s)
- $c_p$: specific heat at constant pressure (kJ/kg K)
- $N_u$: Nusselt number (dimensionless)
- $Pr$: Prandtl number (dimensionless)
- $T$: temperature of the flow field (K)
- $T_w$: temperature at the plate (K)
- $T_\infty$: temperature of the fluid at infinity (K)
- $f$: similarity function (dimensionless)
- $E_a$: the activation energy (kJ/mol)
- $k$: the Boltzmann constant ($k = 8.61 \times 10^{-5}$ eV / K)
- $S_c$: Schmidt number (dimensionless)
- $E$: The non-dimensional activation energy
- $G_r$: Grashof number
- $G_m$: Modified (Solutal) Grashof number (dimensionless)
GREEK SYMBOLS

\( \theta \) : dimensionless temperature, \( \phi \) : dimensionless concentration

\( \eta \) : dimensionless similarity variable, \( \delta \) : Scale factor(m),

\( \nu \) : kinematic viscosity \((m^2/s)\), \( \rho \) : density of the fluid \((Kg. m^{-3})\),

\( \kappa \) : thermal conductivity \((kJ.m^{-1}. S^{-1}. K^{-1})\), \( \tau \) : shear stress \(((kg. m^{-1}. S^{-2}))\),

\( \mu \) : fluid viscosity \((kg/m.s)\), \( \lambda^2 \) : The non-dimensional chemical reaction rate constant

\( \beta_T \) and \( \beta^* \): the coefficients of volume expansions for temperature and concentration respectively \((T^{-1}, C^{-1})\)

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