Soret Effect on the Onset of Convection in a Binary Viscoelastic Fluid Saturated Porous Layer with Internal Heat Source

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Abstract

Internal heating on the onset of Darcy-Brinkman convection in a binary viscoelastic fluid-saturated sparsely packed porous layer with Soret effect is studied using linear stability analyses. The Oldroyd-B model is employed to describe the rheological behavior of binary fluid. An extended form of the Darcy-Oldroyd law incorporating Brinkman’s correction and time derivative is used to describe the flow through a porous layer and the Oberbeck-Boussinesq approximation is invoked. The onset criterion for stationary and oscillatory is derived analytically. The effect of internal Rayleigh number, Soret parameter, relaxation and retardation parameters, solute Rayleigh number, Darcy number, Darcy-Prandtl number, normalized porosity, and Lewis number on the stability of a system is investigated and is shown graphically.

Keywords: Double-diffusive convection (DDC); Viscoelastic fluid; Porous layer; Internal heat source; Heat mass transfer; Soret parameter

1. Introduction

Double diffusive convection in porous media occurs in many systems, and this problem has attracted considerable interest over the years due to its numerous fundamental and industrial applications, such as high-quality crystal production, liquid gas storage, migration of moisture in fibrous insulation, transport of contaminants in saturated soil, solidification of magmas. Extensive reviews of the literature on this subject can be found in the books by Ingham and Pop (1998, 2005), Vafai (2000, 2005), Nield and Bejan (2006), and Vadasz (2008).

With the growing importance of non-Newtonian fluids in modern technology and also due to their natural occurrence, the investigations on such fluids are quite desirable. In particular, the flow of viscoelastic fluid is of considerable importance in several fields of applications such as material processing, petroleum, chemical and nuclear industries, carbon dioxide-geologic sequestration, bioengineering, and reservoir engineering. Although the problem of Rayleigh...
Benard convection (RBC) has been extensively investigated for Newtonian fluids, relatively little attention has been denoted to the thermal convection of viscoelastic fluids (see eg., Li and Khayat 2005 and references therein).

Internal heat generation becomes very important in geophysics, reactor safety analysis, fire & combustion studies and storage of radioactive materials. The onset of convection in a horizontal layer of an anisotropic porous medium with internal heat source subjected to an inclined temperature gradient was studied by Parthiban and Parthiban (1997). An analytical solution for small Rayleigh number in a finite container with isothermal walls and uniform heat generation within the porous medium was given by Hishi et al (2006). Recently, Bhadauria et al (2011) studied the natural convection in a rotating anisotropic porous layer with internal heat generation using a weak nonlinear analysis. In (2012), Bhadauria studied double diffusive natural convection in an anisotropic porous layer in the presence of internal heat source.

If the cross diffusion terms are included in the species transport equations, then the situation will be quite different. Due to the cross diffusion effects, each property gradient has a significant influence on the flux of the other property. A flux of salt caused by a special gradient of temperature is called the Soret effect. Similarly, a flux of heat caused by a spatial gradient of concentration is called the Dufour effects. The Dufour coefficient is of order of magnitude smaller than the Soret coefficient in liquids, and the corresponding contribution to the heat flux may be neglected. Many studies can be found in the literature concerning the Soret and Dufour effects. A study by Rudraiah and Malashetty (1986) discussed the double diffusive convection in a porous medium in the presence of Soret and Dufour effects. In another study, Rudraiah and Siddheshwar (1998) investigated a weak nonlinear stability analysis of double diffusive convection with cross diffusion in a fluid saturated porous medium. Recently, Gaikwad and Kamble (2012) investigated double diffusive convection in a fluid saturated anisotropic porous layer with Soret effect. Also more recently Altawallbeh et al (2013) have investigated the linear and nonlinear double diffusive convection in a saturated anisotropic porous layer with Soret effect and internal heat source.

It is well known that Darcy's law is not valid for non-Newtonian fluid flows in porous media. Recently, Swamy et al. (2012) studied the onset of Darcy-Brinkman convection in a binary viscoelastic fluid saturated porous layer, where the modified Darcy-Brinkman-Oldroyd model has been developed. This model overcomes not only the shortcomings encountered in the modified Darcy-Oldroyd model but also the disadvantages encountered in the Jeffrey's model. By using a variable porous parameter (Darcy number), the modified Darcy-Brinkman-Oldroyd model bridges the gap between nonporous cases in which $Da \rightarrow \infty$ and very densely packed porous cases in which $Da \rightarrow 0$. A better understanding of the characteristics of the Darcy-Brinkman equation is therefore an important part of more practical problems and thus motivates the present report.

Although some literature on double diffusive convection in a porous medium saturated by ordinary fluid with or without an internal heat source and Soret effect is available, very little attention has been devoted to the study of double diffusive convection in a porous layer saturated by a binary viscoelastic fluid with an internal heat source and Soret effect. A very little attention has been devoted to this study in which double diffusive convection in a sparsely packed porous layer saturated by a binary viscoelastic fluid in the presence of an internal heat source and Soret effect has been investigated. Therefore the objective of the present work is to study the onset of double diffusive convection in a horizontal binary viscoelastic fluid saturated porous layer with an internal heat source in the presence of Soret effect using a modified Darcy-
Brinkman-Oldroyd model. In this paper, we intend to perform the onset criteria for stationary and oscillatory convection considering linear stability analyses.

2. Formulation of the Problem

We consider an infinite horizontal sparsely packed, binary viscoelastic fluid saturated porous layer confined between the planes \( z = 0 \) and \( z = d \), with the vertically downward gravity force \( g \) acting on it. Constant temperatures \( \Delta T + T_0 \) and \( T_0 \) with stabilizing concentrations \( \Delta S + S_0 \) and \( S_0 \) respectively are maintained between the lower and upper surfaces. A Cartesian reference frame is chosen with the origin in the lower boundary and the \( z \)-axis vertically upwards. The modified Darcy–Brinkman–Oldroyd model, which includes the time derivative, is employed as a momentum equation. With the Oberbeck–Boussinesq approximation, the basic governing equations are

\[
\nabla \cdot \mathbf{q} = 0, \\
\left(1 + \frac{\partial}{\partial t}\right) \left( \frac{\rho}{\varepsilon} \frac{\partial q}{\partial t} + \nabla p - \rho g \right) = \left(1 + \frac{\partial}{\partial t}\right) \left( \mu \nabla^2 q - \frac{\mu}{K} q \right), \\
\gamma \frac{\partial T}{\partial t} + (q \cdot \nabla) T = \kappa_s \nabla^2 T + Q(T - T_0), \\
\varepsilon \frac{\partial S}{\partial t} + (q \cdot \nabla) S = \kappa_f \nabla^2 S + D_f \nabla^2 T, \\
\rho = \rho_o \left[1 - \beta_f (T - T_0) + \beta_s (S - S_0)\right].
\]

The constants and variables in the above equations have their usual meaning, as given in the Nomenclature. Further, \( \gamma = (\rho c)_m (\rho c_p)_f \), \( \rho c = (1 - \varepsilon) (\rho c)_s + \varepsilon (\rho c_p)_f \), \( c \) is the specific heat of the solid, \( c_p \) is the specific heat of the fluid at constant pressure, the subscripts \( f, s \) and \( m \) denote fluid, solid and porous medium values respectively.
2.1 Basic State

The basic state of the fluid is assumed to be quiescent and is given by

\[ \mathbf{q} = \mathbf{q}_b + \mathbf{q}', \quad T = T_b + T', \quad S = S_b + S', \quad p = p_b + p', \quad \rho = \rho_b + \rho', \]

(6)

Using Eq. (6) into Eqs. (1) – (5) yields

\[ \frac{dp_b}{dz} = -\rho_b g, \quad \kappa_r \frac{d^2 T_b}{dz^2} + Q(T_b - T_o) = 0, \quad \frac{d^2 S_b}{dz^2} = 0, \quad \rho_b = \rho_o \left[ 1 - \beta_r \right], \quad (T_b - T_o) + \beta_s (S_b - S_o) \]

(7)

Then the conduction state temperature and concentration are given by

\[ T_b(z) = \frac{\Delta T \text{Sin} \left( \sqrt{Ri} \left( 1 - \frac{z}{d} \right) \right)}{\text{Sin} \sqrt{Ri}} + T_o, \quad S_b = \Delta S \left( 1 - \frac{z}{d} \right) + S_o. \]

(8)

2.2 Perturbed state

On the basic state, we superpose small perturbations in the form

\[ \mathbf{q} = \mathbf{q}_b + \mathbf{q}', \quad T = T_b + T', \quad S = S_b + S', \quad p = p_b + p', \quad \rho = \rho_b + \rho', \]

(9)

where the primes indicate perturbations. Substituting Eq. (9) into Eqs. (1) – (5) and using the Eqs. (6) – (8), the perturbations are given in the form

\[ \nabla \cdot \mathbf{q}' = 0, \]

(10)

\[ \left( 1 + \frac{\gamma}{c} \frac{\partial}{\partial t} \right) \left( \rho_o \frac{\partial \mathbf{q}}{\partial t} \right) + \nabla p' + \rho_o (\beta_r T' - \beta_s S') g = \left( 1 + \frac{\gamma}{c} \frac{\partial}{\partial t} \right) \left( \mu_r \nabla^2 \mathbf{q} - \frac{\mu}{K} \mathbf{q} \right), \]

(11)

\[ \gamma \frac{\partial T'}{\partial t} + (\mathbf{q} \cdot \nabla) T' + w \frac{\partial T_b}{\partial z} = \kappa_r \nabla^2 T' + QT', \]

(12)

\[ \varepsilon \frac{\partial S'}{\partial t} + (\mathbf{q} \cdot \nabla) S' + w \frac{\partial S_b}{\partial z} = \kappa_s \nabla^2 S' + D \nabla^2 T', \]

(13)

\[ \rho' = - \rho_o \left[ \beta_r T' - \beta_s S' \right]. \]

(14)

By operating curl twice on Eq. (11) we eliminate \( p' \) and then use the scaling

\[ (x', y', z') = (x, y, z) d, \quad (u', v', w') = (\kappa_r / d) (u, v, w), \quad t' = t \left( \gamma d^2 / \kappa_r \right), \quad T' = (\Delta T) T^*, \quad S' = (\Delta S) S^*, \]

(15)

to non-dimensionalize Eqs. (10) – (14) in the form (on dropping the asterisks),
where \( Ri = \frac{Qd^2}{\kappa_T} \) is the internal Rayleigh number, and all other non-dimensional parameters are as defined in the Nomenclature. The thermal Rayleigh number characterizes the buoyancy due to the thermal gradient and that due to the solute gradient is indicated by the solute Rayleigh number. The viscoelastic character of the liquid mixture appears in the relaxation parameter \( \lambda_1 \) (which is also known as the Deborah number) and retardation parameter, \( \lambda_2 \). The Deborah number is a ratio of the relaxation time of the material to a characteristic time of the process. The parameter \( \lambda_2 = 0 \) for a Maxwell fluid while \( \lambda_1 = \lambda_2 = 0 \) for a Newtonian fluid. For dilute polymeric solutions the value of Deborah number is most likely in the range \([0.1, 2]\) and \( \lambda_2 \) in the range \([0.1, 1]\). It is worth mentioning here that the Darcy-Prandtl number \( Pr_D \) includes the Prandtl number, Darcy number, porosity and the specific heat ratio. It depends on the properties of the fluid and on the nature of the porous matrix. The Prandtl number affects the stability of the porous system through this combined dimensionless group. For the sparse porous media, \( Da \in [10^{-2}, 1] \), \( \varepsilon \approx 0.5 \) and a typical value for the Prandtl number for a viscoelastic fluid is \( Pr = 10 \). Since \( Pr_D \) is magnified by a factor \( \varepsilon^{-1} Da \) the reasonable range for \( Pr_D \) will be \([5, 500]\). The ratio between thermal and solute diffusivities is characterized by the Lewis number and can be varied in the range of \([1, 1000]\). The normalized porosity \( \varepsilon_n \) is expressed in terms of the porosity of the porous medium \( \varepsilon \) and the solid to fluid heat capacity ratio, \( \gamma \). Since \( 0 < \varepsilon < 1 \), it is clear that \( 0 < \varepsilon_n < 1 \).

Since the boundaries are assumed to be stress free, isothermal and isohaline; the Eqs. (16) – (18) are to be solved for the boundary conditions

\[
\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \left[ \frac{1}{Pr_D} \frac{\partial}{\partial t} \nabla^2 w - Ra_r \nabla^2 T + Ra_s \nabla^2 S \right] - \left(1 + \lambda_2 \frac{\partial}{\partial t}\right) \left( Da \nabla^4 w - \nabla^2 w \right) = 0, \quad (16)
\]

\[
\left[ \frac{\partial}{\partial t} - \nabla^2 - Ri \left( q \cdot \nabla \right) \right] T - w = 0, \quad (17)
\]

\[
\left[ \varepsilon_n \frac{\partial}{\partial t} - \frac{1}{Le} \nabla^2 + (q \cdot \nabla) \right] S - S_r \nabla^2 T - w = 0, \quad (18)
\]

3. Linear Stability Analysis

In this section we predict the thresholds of both marginal and oscillatory convections. The Eigenvalue problem defined by Eqs. (16) – (18) and subjected to the boundary conditions (19) is solved using the time-dependent periodic disturbances in a horizontal plane. Assuming that the amplitudes of the perturbations are very small, we write
\[
\begin{pmatrix}
W \\
T \\
S
\end{pmatrix} = \begin{pmatrix} W(z) \\
\Theta(z) \\
\Phi(z) \end{pmatrix} \exp\left[i(lx + my) + \sigma t \right].
\]

(20)

Where \( l, m \) are horizontal wavenumbers and \( \sigma \) is the growth rate. Infinitesimal perturbations of the rest state may either dampen or grow depending on the value of the parameter \( \sigma \). Substituting Eq. (20) into the linearized version of Eqs. (16) – (18), we obtain

\[
(1 + \lambda \sigma ) \left[ \frac{\sigma}{Pr_D} \left( D^2 - \alpha^2 \right) W + Ra_r a^2 \Theta - Ra_s a^2 \Phi \right] - (1 + \lambda \sigma ) \left( D^2 - \alpha^2 \right) \left[ Da \left( D^2 - \alpha^2 \right) - 1 \right] W = 0,
\]

(21)

\[
\left[ \sigma - \left( D^2 - \alpha^2 \right) - Ri \right] \Theta - W = 0,
\]

(22)

\[
\left[ \varepsilon_n \sigma - Le^{-1} \left( D^2 - \alpha^2 \right) \right] \Phi - W - S_r \left( D^2 - \alpha^2 \right) \Theta = 0,
\]

(23)

where \( D \equiv d/dz \) and \( \alpha^2 = l^2 + m^2 \). The boundary conditions (19) now becomes

\[
W = D^2 W = \Theta = \Phi = 0 \text{ at } z = 0, 1.
\]

(24)

We assume the solutions in the form

\[
(W(z), \Theta(z), \Phi(z)) = (W_0, \Theta_0, \Phi_0) \sin n\pi z, (n = 1, 2, 3, \ldots).
\]

(25)

The most unstable mode corresponds to \( n = 1 \) (fundamental mode). Therefore, substituting Eqs. (25) with \( n = 1 \) into Eqs. (21) – (23), we obtain a matrix equation of the form,

\[
\begin{pmatrix}
\frac{\sigma}{Pr_D} + \Lambda \left( Da \delta^2 + 1 \right) \\
-1 \\
-1
\end{pmatrix} \delta^2 - a^2 Ra_r a^2 Ra_s \\
\begin{pmatrix} W_0 \\
\Theta_0 \\
\Phi_0 \end{pmatrix} = \begin{pmatrix} 0 \\
0 \\
0 \end{pmatrix},
\]

(26)

where \( \delta^2 = \pi^2 + a^2 \), \( \Lambda = \frac{1 + \lambda \sigma \omega}{1 + \lambda \omega} \). For the above matrix Eq. (26) to have the nontrivial solution, we require

\[
Ra_r = \frac{\left( \sigma + \delta^2 - Ri \right)}{a^2} \left( \delta^2 \left[ \frac{\sigma}{Pr_D} + \Lambda \left( Da \delta^2 + 1 \right) \right] + \frac{Ra_s}{\varepsilon_n \sigma + \delta^2 Le^{-1}} \right) = \frac{Ra_s S_r \delta^2}{\left( \varepsilon_n \sigma + \delta^2 Le^{-1} \right)}.
\]

(27)
3.1 Marginal state

For the validity of the principle of exchange of stabilities (i.e., steady case), we have $\sigma = 0$ (i.e., $\sigma_r = \sigma_i = 0$) at the margin of stability. Then the Rayleigh number at which the marginally stable steady mode exists becomes

$$Ra_r^{st} = \left(\delta^2 - Ri\right) \left[\frac{\sigma^2}{a^2} \left(Da\delta^2 + 1\right) + \frac{Le Ra_s}{a^2 \delta^2}\right] - \frac{Ra_s Le S}{a^2}.$$  \hspace{1cm} (28)

The minimum value of the Rayleigh number $Ra_r^{st}$ occurs at the critical wavenumber $a = a_r^{st}$ where $a_r^{st} = \sqrt{s}$ satisfies the equation

$$k_s s^2 + k_2 s^4 + k_3 s^6 + k_4 s^8 + k_5 s + k_6 = 0,$$  \hspace{1cm} (29)

Where,

$$k_1 = 2 Da, \quad k_2 = 1 + 7 Da\pi^2 - Da R_i,$$
$$k_3 = 2\pi^2 \left(1 + 4\pi^2 Da - Da R_i\right), \quad k_4 = 2 Da\pi^6 + \pi^2 R_i + Le \left(-1 + S_r\right) Ra_s,$$
$$k_5 = -2\pi^4 \left(1 + Da\pi^2\right) \left(\pi^2 - R_i\right) + 2 Le \left(\pi^2 \left(-1 + S_r\right) + R_i\right) Ra_s,$$
$$k_6 = -\pi^6 \left(1 + Da\pi^2\right) \left(\pi^2 - R_i\right) + Le\pi^2 \left(\pi^2 \left(-1 + S_r\right) + R_i\right) Ra_s.$$  \hspace{1cm} (30)

In the absence of a heat source for the sparsely packed porous medium (i.e., when $R_i = 0$) Eq. (28) reduces to

$$Ra_r^{st} = \frac{1}{a^2} \left[\delta^4 \left(Da\delta^2 + 1\right) + Le Ra_s \left(1 - S_r\right)\right].$$  \hspace{1cm} (31)

In the absence of Soret parameter (i.e., when $S_r = 0$) the above Eq. (31) reduces to

$$Ra_r^{st} = \frac{1}{a^2} \left[\delta^4 \left(Da\delta^2 + 1\right) + Le Ra_s\right].$$  \hspace{1cm} (32)

In the absence of Solute Rayleigh number (i.e., when $Ra_s = 0$) the above Eq. (32) reduces to

$$Ra_r^{st} = \frac{\delta^4 \left(Da\delta^2 + 1\right)}{a^2}.$$  \hspace{1cm} (33)

When $Da \rightarrow 0$, that is for densely packed porous medium Eq. (33) becomes

$$Ra_r^{st} = \frac{\delta^4}{{\pi^2}} \left(\frac{\pi^2 + a^2}{a^2}\right)^2,$$  \hspace{1cm} (34)

which has the critical value $Ra_c^{st} = 4\pi^2$ for $a_r^{st} = \pi$ obtained by Horton and Rogers (1945) and Lapwood (1948).

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3.2 Oscillatory state

We now set $\sigma = i w_i$ in Eq. (27) and clear the complex quantities from the denominator, to obtain

$$Ra_f = \Delta_1 + i \omega_1 \Delta_2,$$

(35)

Where,

$$\Delta_1 = \left[ -\delta^2 (w^2 + \lambda_i^2 w^4 + (1 + Da \delta^2) Pr_D (\delta^2 - R_i + w^2 \lambda_i + w^2 (1 + (\delta^2 - R_i) \lambda_i)) \right]$$

$$+ \left[ \frac{LeRa_s ((1 + S_r) \delta^2 - \delta^2 R_i + Le \omega_n \varepsilon_n)}{\delta^4 + Le \omega_n w^2 \varepsilon_n^2} \right],$$

$$\Delta_2 = (Le^2 \delta^2 \varepsilon_n \lambda_i (\delta^2 - R_i) \lambda_i + (1 + Da \delta^2) Pr_D (R_i \lambda_i) + (Le^2 Pr_D (-1 + S_r) \delta^2 + R_i) \lambda_i + (1 + Da \delta^2)(1 + (\delta^2 - R_i) \lambda_i)),$$

Since $Ra_f$ is a physical quantity, it must be real. Hence, from Eq. (35) it follows that either $\omega_i = 0$ (steady onset) or $\Delta_2 = 0$ ($\omega_i \neq 0$, oscillatory onset). For oscillatory onset, setting $\Delta_2 = 0$ ($\omega_i \neq 0$) gives an expression for frequency of oscillations in the form (dropping the subscript $i$)

$$a_0 \left( \omega^2 \right)^2 + a_1 \omega^2 + a_2 = 0,$$

(36)

where,

$$a_0 = \left[ \frac{Le^2 \delta^2 \varepsilon_n \lambda_i (\delta^2 - R_i) \lambda_i + (1 + Da \delta^2) Pr_D \lambda_i}{} \right],$$

$$a_1 = \left[ (le^2 Pr_D (-1 + S_r) \delta^2 + R_i) Ra_s \varepsilon_n \lambda_i + \delta^2 \lambda_i ((\delta^2 - \delta^2 R_i + Le \omega_n \varepsilon_n) \lambda_i) + (1 + Da \delta^2) Pr_D R_i \lambda_i \right],$$

$$a_2 = \left[ (\delta^2 - \delta^2 R_i + Pr_D R_i \delta^2 + Le ((1 + S_r) \delta^2 + R_i) \varepsilon_n) \lambda_i + (1 + Da \delta^2)(1 + (\delta^2 - R_i) \lambda_i) \right].$$

Now Eq. (35) with $\Delta_2 = 0$, gives
The analytical expression for the oscillatory Rayleigh number given by Eq. (37) is minimized with respect to the wavenumber numerically, after substituting for $\omega^2 (> 0)$ from Eq. (36), for various values of physical parameters in order to know their effects on the onset of oscillatory convection.

4. Results and Discussion

Internal heating on the onset of Darcy-Brinkman convection in a binary viscoelastic fluid-saturated sparsely packed porous layer with Soret effect is studied using linear stability analyses. The expressions for stationary and oscillatory Rayleigh numbers are derived analytically and the graphs are plotted for different values of different parameters and the results are discussed here.

The neutral stability curves in the $(Ra_f - a)$ plane for various parameter values are as shown in Figs.1-9. We fixed the values for the parameters except the varying parameter.

Fig.1. Neutral stability curves for different values of $S_r$.  

Fig.2. Neutral stability curves for different values of $\epsilon_n$.  

$$Ra_f^{osc} = \left[ \left( \frac{-\delta^2(w^2 + \lambda_1^2 w^4 + (1 + Da\delta^2)Pr_{D}(\delta^2 - R_s + w^2\lambda_1 + w^2(-1 + (\delta^2 - R_s^2)\lambda_1\lambda_2)))}{Pr_{D}(1 + w^2\lambda_1^2)} \right) + \frac{LeRa_s((1 + S_r)\delta^4 - \delta^2 R_s + Le\omega^2\epsilon_n)}{\delta^4 + Le^2 w^2 \epsilon_n^2} \right].$$  

(37)
In Fig. 1, the neutral stability curves for different values of Soret parameter $S_r$ are drawn. From this figure, we observe that with an increase in the value of negative Soret parameter, the stationary Rayleigh number increases, indicating that the negative Soret number stabilizes the system. On the other hand, for positive Soret parameter, the minimum of the stationary Rayleigh number decreases with an increase of the Soret parameter, indicating that the positive Soret parameter destabilizes the system in the stationary mode. In the case of oscillatory mode, we find that the negative Soret coefficient has destabilizing effect whereas the positive Soret coefficient has a stabilizing effect.

The effect of normalized porosity parameter on the neutral curves is shown in Fig. 2. We observe that the effect of increasing the normalized porosity parameter decreases the minimum of the Rayleigh number, indicating that the effect of normalized porosity parameter is to advance the onset of double diffusive convection. Further, it is important to note that the effect of normalized porosity parameter is significant for small $\varepsilon_n$.

Fig. 3 depicts the effect of solute Rayleigh number $Ra_S$ on the neutral stability curves for stationary and oscillatory modes. We find that the effect of increasing $Ra_S$ is to increase the value of the Rayleigh number for stationary mode while decreases for oscillatory mode.

Fig. 4 depicts the neutral stability curves for different values of Lewis number $Le$ are drawn. It is observed that with the increase of $Le$ the critical values of Rayleigh number and the corresponding wavenumber for the overstable mode decrease while those for stationary mode increases. Therefore, the effect of $Le$ is to advance the onset of oscillatory convection whereas its effect is to inhibit the stationary onset.

![Fig. 3. Neutral stability curves for different values of $Ra_S$.](image1)

![Fig. 4. Neutral stability curves for different values of $Le$.](image2)

The neutral stability curves for different values of Darcy-Prandtl number $Pr_D$ are presented in the above Fig. 5. The point where the overstable solution bifurcates into the stationary solution is observed to be shifted towards a higher value of $a$ with the increasing $Pr_D$. From this figure it is evident that for small and moderate values of $Pr_D$, the critical value of oscillatory Rayleigh
number decreases with the increase of $Pr_D$, however this trend is reversed for large values of $Pr_D$.

Fig. 5. Neutral stability curves for different values of $Pr_D$.

Fig. 6. Neutral stability curves for different values of $Da$.

Fig. 6 exhibits the effect of Darcy number $Da$ on the neutral stability curves for the fixed values of other governing parameters. It is observed that the oscillatory convection sets in prior to the stationary convection for the values of the parameters chosen for this figure. We find that with the increase in the values of Darcy number $Da$ the critical Rayleigh number and the corresponding wavenumber for the stationary and oscillatory mode increases. Therefore, the effect of Darcy number $Da$ is to stabilize the system.

Fig. 7. Neutral stability curves for different values of $R_i$.

Fig. 8. Neutral stability curves for different values of $\lambda_1$.
Fig. 7 shows the neutral stability curves for different values of the internal Rayleigh number \( R_i \) and for fixed values of other parameters. We observe from this figure that the minimum value of the Rayleigh number for both stationary and oscillatory modes decreases with an increase in the value of the internal Rayleigh number \( R_i \), indicating that the effect of the internal Rayleigh number is to destabilize the system.

Fig. 8 depicts the effect of relaxation parameter \( \lambda_1 \) on the neutral stability curves for oscillatory mode. We find that the minimum of the oscillatory Rayleigh number decreases with increasing values of \( \lambda_1 \), indicating that the effect of relaxation parameter is to advance the onset of double diffusive convection. It is important to note that when \( \lambda_1 = \lambda_2 \) the oscillatory neutral curve concur with that for the viscous Newtonian case. That is, the stability of the system in the oscillatory mode becomes independent of the viscoelastic parameters when their values become the same. Further, we observe that the minimum of the oscillatory Rayleigh number shift toward the smaller values of the wavenumber with increasing \( \lambda_1 \) indicating that the cell width increases with increasing \( \lambda_1 \).

The influence of retardation parameter \( \lambda_2 \) on the oscillatory neutral curves is revealed in Fig. 9. We observe that increasing of the retardation parameter increases the minimum of the Rayleigh number, indicating that the effect of retardation parameter stabilizes the system towards the oscillatory mode. This figure also indicates that the neutral curve is identical with that of the Newtonian case when \( \lambda_2 = \lambda_1 \). Further, we find that the minimum Rayleigh number shift toward the larger values of the wavenumber with increasing \( \lambda_2 \) indicating that the cell width decreases with increasing \( \lambda_2 \).

5. Conclusions

The onset of Darcy-Brinkman convection in a horizontal, sparsely packed porous layer saturated with a binary viscoelastic fluid with an internal heat source and Soret effect is studied.
analytically using linear stability theory. The usual normal mode technique is used to solve the problem. The following important conclusions are drawn: The effect of Soret parameter destabilizes the system in the stationary mode while destabilizing the system in oscillatory case. The effect of internal Rayleigh number is to destabilize the system for stationary and oscillatory modes. The effect of normalized porosity is to advance the onset of double diffusive convection and it is significant for small value. Effect of the solute Rayleigh number to delay both the stationary and oscillatory modes. The effect of Lewis number is to advance the onset of oscillatory convection whereas its effect is to inhibit the stationary onset. The effect of Darcy number is stabilizing the system for stationary and oscillatory modes. The effect of relaxation parameter is to advance the onset of double diffusive convection. The stability of the system in oscillatory mode becomes independent of the viscoelastic parameters when the values of relaxation and retardation parameters become same. The effect of retardation parameter stabilizes the system towards the oscillatory mode.

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