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Research Article

Finite Element Method and Müller-Breslau Principle in Influence Line of Rigid Frame Bridges

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Abstract

This paper proposes a Finite Element Method (FEM) considering beam elements of different cross-sections to analyze a rigid frame bridge. Slope-deflection method ignoring beam element axial deformation (simplified FEM) was applied to a rigid frame bridge example first. Then Slope-deflection method considering beam axial deformation (fully implemented FEM) was applied to the same example. Results indicate that the girder horizontal displacement calculated with slope-deflection method ignoring beam axial deformation is a little larger. At last, the results of the influence line of bending moment were validated with Müller-Breslau principle. Influence line for moment from slope-deflection method coincides with the result from Müller-Breslau Principle.

Keywords: Rigid frame bridge; Fully implemented FEM; Influence line; Müller-Breslau principle

1. Introduction

There are many rigid frame bridges in the world.Assistant Shipping Channel Bridge of Humen Bridge is one of them (Shao, 2005). According to the PAHBI, ''The 54'-long and 37' wide, reinforced concrete, rigid frame bridge (Fig.1) built in 1938 has flared wing walls and is finished with Moderne-style balustrades (BridgeMapper, 2011). This paper intends to determine the influence line of a rigid frame bridge.

Influence line is to investigate the live load effect on the structure. The influence line can determine the support reaction or the internal force in the beam. The live load may generate from the moving vehicles (Leet, 2008, Institute of Bridge Science of Bridge Engineering Department of Railways Ministry, 1996, Li and Wu, 2005, Niu and Wang, 2011). Live load includes moving uniform load or concentrated load. Tatjana Grigorjeva et al. (2008) presents a summary of numerical analysis on

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static behaviour of suspension bridges with varying rigidity of cables (Grigorjeva, et al., 2008). Based on railway transport Shawan Bridge in Guangzhou City, a continuous rigid frame bridge was analyzed by commercial finite element method (Qi, et al., 2007).

Fig.1. Stoystown Road Overpass (BridgeMapper, 2011)

Finite element method originated from slope-deflection method. In fully implemented finite element method, the beam element has 3 DOF at each node : the axial displacement, the transverse displacement and a rotation. Another kind of the slope-deflection method (could be called as simplified FEM) is the slope-deflection method not considering beam axial deformation induced by axial internal load (Shan, 2011, Zhi, 1985).

However little work has been done on: influence of considering or not considering beam axial deformation on the results of internal load and node displacement of rigid frame bridge as in Fig.2 (Li, 2007) .

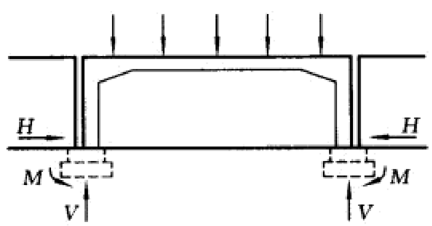

Fig.2. Single span rigid frame bridge (Li, 2007)

In the following example, a rigid frame bridge was first analyzed with FEM not considering beam axial deformation, and then considering beam axial deformation. Bending moments were determined when a concentrated unit load moves along the bridge girder. Functions were used to express influence line of bending moment. The two methods were then compared.
At last, results of the influence line of bending moment were validated with Müller-Breslau principle.

2. A Single Span Rigid Frame Bridge Example

For rigid frame in Fig.3, according to reference (Shao, 2005) and (Liu, 1957), assume parameters are as following: Pillar height h=7.5m, girder length $l = 10m$, unit live load P=1N. The bridge is built with homogeneous material with elastic modulus $E = 3 \times 10^4 MPa$.

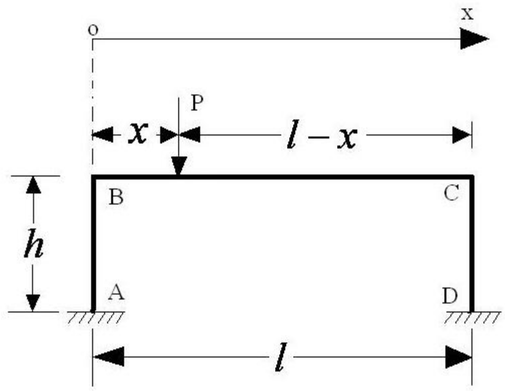

Fig. 3. A single span rigid frame bridge with unit live load P (P=1N) moving on the girder

Pillar: Section width b'=1.024m; section height h'=0.586m. The area moment of inertia of Pillar I=17.15*10-3 m^4; Flexural rigidity $(EI)_h$ =EI=17.15*10-3 m4*3*1010Pa=5.145*108 N*m^2
Cross-section area $(A)_h$ =b'*h'=0.6 m^2.

Girder: Section width b''=0.351m; section height h''=1.195m. The area moment of inertia of girder $I = \frac{1}{12} b'' h''^3 = \frac{1}{12} * 0.6 * 1 = 0.05$ m^4; Flexural rigidity $(EI)_l$ =EI=0.05m4*3*1010Pa=1.5*109 N*m^2.
Cross-section area $(A)_l$ =b''*h''=0.42 m^2.

3. Slope-deflection Method Ignoring Bar Axial Deformation (Simplified FEM)

To determine influence line of moment M_{BA} which is the moment at the end of beam segment at joint B, the slope-deflection method should be used when concentrated unit load P moves along beam segment BC in Figure 3. This paper determined the influence line for M_{BA} when unit live load P is at beam BC, and express it with function.

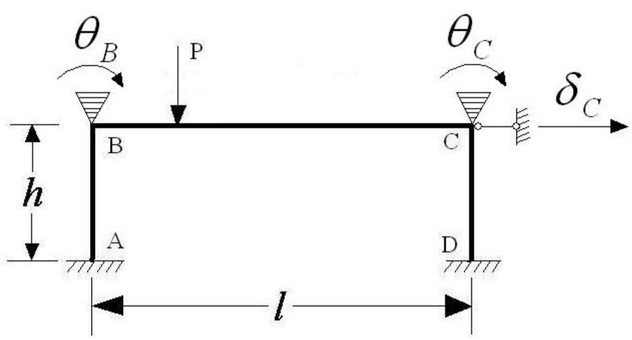

Fig. 4. Fundamental structural system of the structure in Fig.2

Fundamental structural system ignoring bar BC axis deformation of the structure in Fig.3 was set as in Fig.4. Define slope θ is positive when it is clockwise and negative when it is counterclockwise. Define moment acted on the beam segment end is positive when it is clockwise and negative when it is counterclockwise. Define deflection δ is positive when the bar BC moves towards positive direction of the x axis and negative when bar BC moves towards the negative direction of the x axis. Define the horizontal shear force acted on the beam segment end is positive when its direction is towards the positive direction of the x axis and negative when its direction is towards the negative direction of the x axis.

It is obvious that 0<x<l in Fig.3.

With slope-defection method, following equation can be given given as

$$\frac{P}{l^2}x(l-x)^2-\left(4\frac{(EI)_l}{l}+4\frac{(EI)_h}{h}\right)\theta_B-2\frac{(EI)_l}{l}\theta_c+6\frac{(EI)_h}{h^2}\delta_c=0 \quad (1)$$

$$-\frac{P}{l^2}x^2(l-x)-2\frac{(EI)_l}{l}\theta_B-\left(4\frac{(EI)_l}{l}+4\frac{(EI)_h}{h}\right)\theta_c+6\frac{(EI)_h}{h^2}\delta_c=0 \quad (2)$$

$$0+6\frac{(EI)_h}{h^2}\theta_B+6\frac{(EI)_h}{h^2}\theta_c-24\frac{(EI)_h}{h^3}\delta_c=0 \quad (3)$$

Define:

$$\left(4\frac{(EI)_l}{l}+4\frac{(EI)_h}{h}\right)=A \quad (4)$$

$$2\frac{(EI)_l}{l}=B \quad (5)$$

$$6\frac{(EI)_h}{h^2}=C \quad (6)$$

$$\frac{P}{l^2}x(l-x)^2=D \quad (7)$$

$$-\frac{P}{l^2}x^2(l-x)=E \quad (8)$$

$$24\frac{(EI)_h}{h^3}=F \quad (9)$$

Solving Equations from (1) to (9) simultaneously gives:

$$\theta_C = \frac{\left(B - C*\frac{C}{F}\right)*D - \left(A - C*\frac{C}{F}\right)*E}{\left(\left(B - C*\frac{C}{F}\right)*\left(B - C*\frac{C}{F}\right) - \left(A - C*\frac{C}{F}\right)*\left(A - C*\frac{C}{F}\right)\right)} \tag{10}$$

$$\theta_B = \frac{1}{\left(B - C*\frac{C}{F}\right)}\left(E - \left(A - C*\frac{C}{F}\right)*\theta_C\right) \tag{11}$$

$$\delta_c = \frac{C}{F}*\theta_B + \frac{C}{F}*\theta_c \tag{12}$$

Internal load moment acted on the beam segment end at support B is:

$$M_{BA} = -4\frac{(EI)_h}{h}\theta_B + 6\frac{(EI)_h}{h^2}\delta_c \tag{13}$$

$$M_{BC} = \frac{P}{l^2}x(l-x)^2 - \frac{4(EI)_l}{l}\theta_B - \frac{2(EI)_l}{l}\theta_c \tag{14}$$

4. Slope-deflection Method Considering Bar Axial Deformation (Fully implemented FEM)

Define slope θ_B and θ_c are positive when they are clockwise. Define horizontal displacements u_B and u_c are positive when node B and C moves toward x axis positive direction. Define vertical displacements v_B and v_C are positive when they cause the compression of bar AB and bar CD respectively.

With variables $\theta_B, u_B, v_B, \theta_c, u_c, v_C$ in Fig.5, and with slope-deflection method considering bar axial deformations, equations below could be established as:

$$\frac{P}{l^2}x(l-x)^2 - \left(4\frac{(EI)_h}{h} + 4\frac{(EI)_l}{l}\right)\theta_B - 2\frac{(EI)_l}{l}\theta_c + 6\frac{(EI)_h}{h^2}u_B - 6\frac{(EI)_l}{l^2}v_B + 6\frac{(EI)_l}{l^2}v_C = 0 \tag{15}$$

$$6\frac{(EI)_h}{h^2}\theta_B + E(A)_l\frac{1}{l}u_c - E(A)_l\frac{1}{l}u_B - 12\frac{(EI)_h}{h^3}u_B = 0 \tag{16}$$

$$6\frac{(EI)_l}{l^2}\theta_B + 6\frac{(EI)_l}{l^2}\theta_C + 12\frac{(EI)_l}{l^3}v_B + E(A)_h\frac{1}{h}v_B - 12\frac{(EI)_l}{l^3}v_C - \frac{P(l-x)^2(l+2x)}{l^3} = 0 \tag{17}$$

$$-\frac{P}{l^2}x^2(l-x)-2\frac{(EI)_l}{l}\theta_B-\left(4\frac{(EI)_l}{l}+4\frac{(EI)_h}{h}\right)\theta_C+6\frac{(EI)_h}{h^2}u_C-6\frac{(EI)_l}{l^2}v_B+6\frac{(EI)_l}{l^2}v_C=0 \quad (18)$$

$$6\frac{(EI)_h}{h^2}\theta_C-\left(E(A)_l\frac{1}{l}+12\frac{(EI)_h}{h^3}\right)u_C+E(A)_l\frac{1}{l}u_B=0 \quad (19)$$

$$-6\frac{(EI)_l}{l^2}\theta_B-6\frac{(EI)_l}{l^2}\theta_C-12\frac{(EI)_l}{l^3}v_B+12\frac{(EI)_l}{l^3}v_C+E(A)_h\frac{1}{h}v_C-\frac{Px^2(l+2(l-x))}{l^3}=0 \quad (20)$$

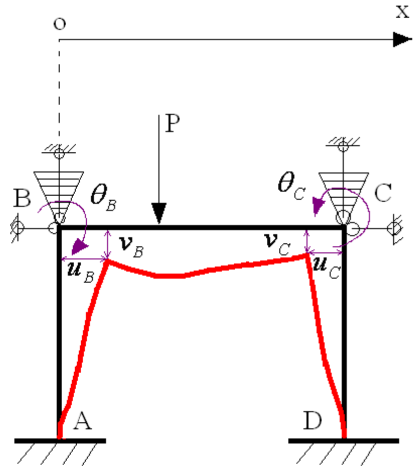

Fig. 5. Slope-deflection method considering bar axial deformation

5. Results

5.1 Comparison between results of simplified FEM and fully implemented FEM

Horizontal displacement at bar BC、slope at B、slope at C and Moment MBA were presented in Figure 6、7、8 and 9.

In Fig.6, horizontal displacement of node B or C were determined with slope-deflection method considering bar axial deformation. In Fig.6, horizontal displacement of bar BC was determined with slope-deflection method not considering bar axial deformation.

With Figure 6、7、8 and 9, it can be concluded that considering or not considering bar axial deformation does not affect the results too much. However slope-deflection method not considering bar axial deformation could not give the displacement of node point like B and C.

Results indicate that the girder horizontal displacement calculated with slope-deflection method not considering bar axial deformation is a little larger than the horizontal displacements of the girder two end points with slope-deflection method considering bar axial deformation. Slope of the node and the calculated internal moment at the node is similar between the results of the two methods.

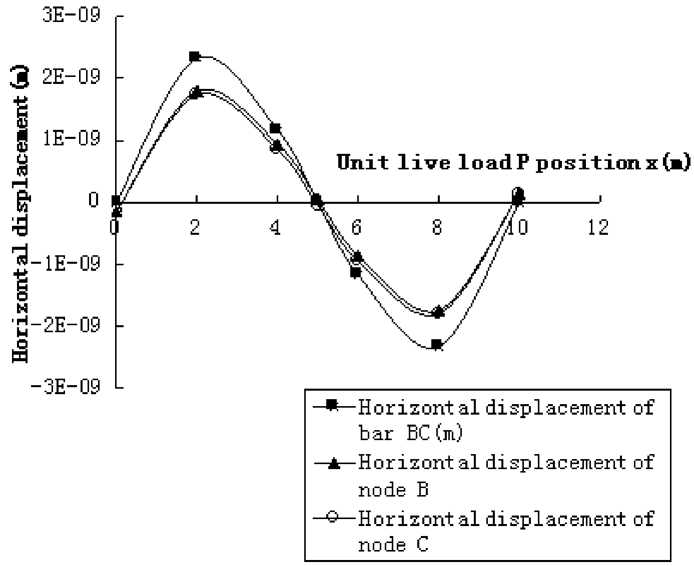

Fig. 6. Horizontal displacement of bar BC

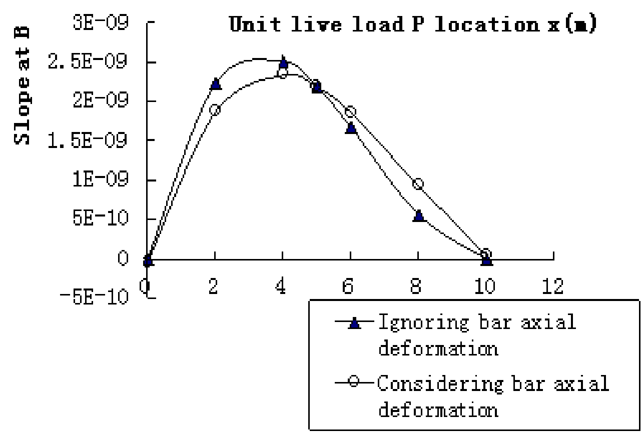

Fig. 7. Slope at B

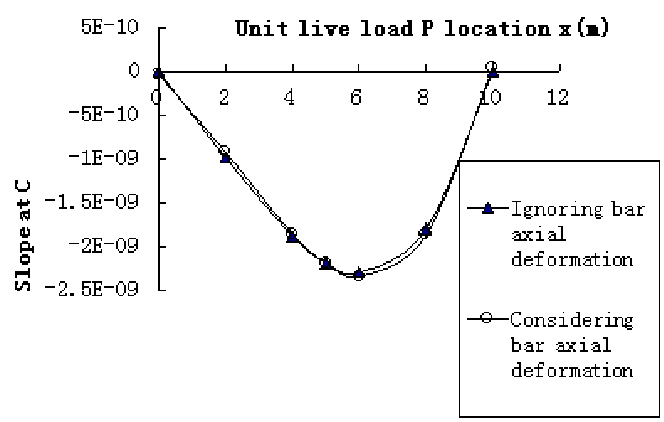

Fig. 8. Slope at C

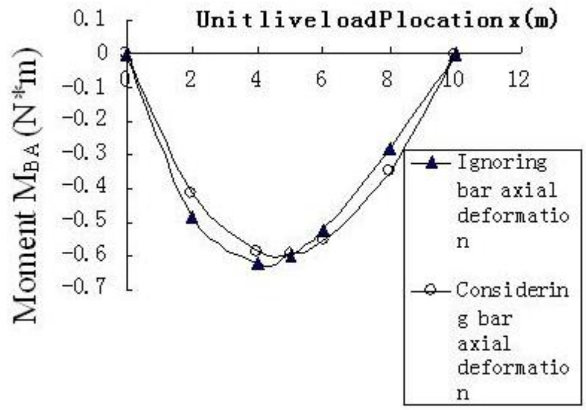

Fig. 9. Moment M_{BA} (N*m)

5.2 Axial displacements with slope-deflection method considering bar axial deformation

Slope-deflection method considering bar axial deformation could give the displacement of node point like B and C. The vertical displacements of B and C were presented as in Figure 10 and 11.

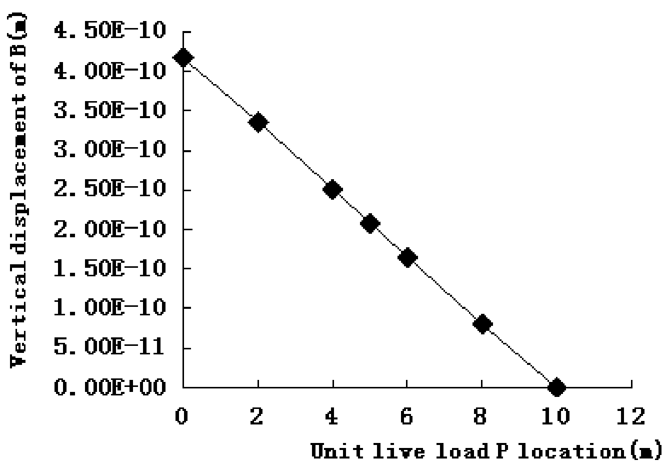

Fig. 10. Vertical displacement of B

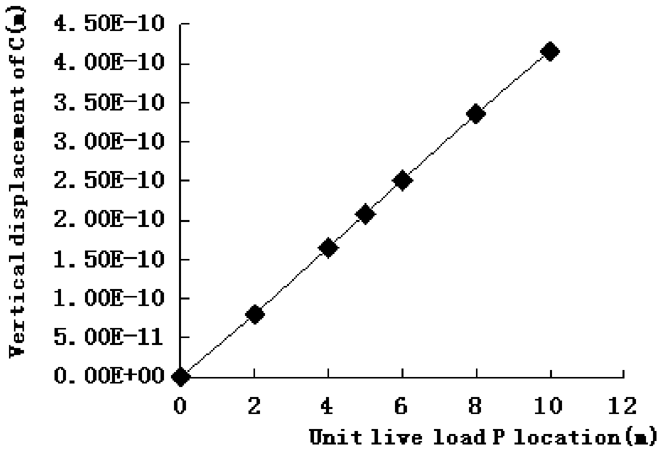

Fig. 11. Vertical displacement of C

6. Validation of Müller-Breslau Principle

According to Müller-Breslau Principle in Ref (Fanous, 2000), influence line for moment at B of Fig.3 can be drawn in the following Fig.12.

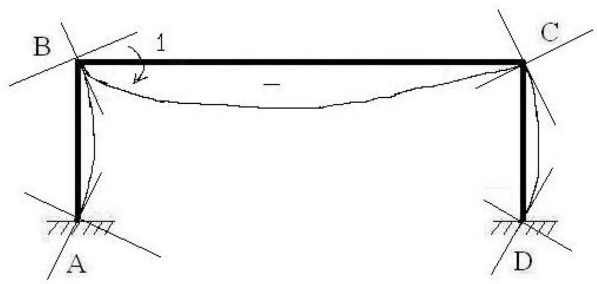

Fig. 12. Influence line for moment at B according to Müller-Breslau Principle

After comparison between Fig.9 and Fig.12, conclusion can be drawn that influence line for moment at B from slope-deflection method coincides with the result from Müller-Breslau Principle.

7. Discussions

7.1 Application of influence lines into practical design

This result facilitates the design of bridges (Heins and Lawrie, 1999) and buildings (Leet, et al., 2008, Hibbeler, 1985). Because dead load, live uniform load and wind load can be determined with American codes or specifications (Leet, et al., 2008, Heins and Lawrie, 1999), but the live concentrated load is a complex and rough problem. The influence line gives the maximum moment at joint B when the live load is moving long the length direction of bridge. It also gives the live load

P position when M_{BA} reaches its maximum bending moment value. These values can be used to design the concrete bridge (Winter and Nilson, 1979).

7.2 Comparison between the two methods and the results of the two methods

It can be concluded that considering or not considering bar axial deformation does not affect the results too much. However slope-deflection method not considering bar axial deformation could not give the displacement of node point. However the slope-deflection method considering bar axial deformation is more complex than the slope-deflection method not considering bar axial deformation.

7.3 Limitations of the methods in this paper

Methods in this paper only considered the difference of the inertia area moment between pillars and girders in a FEM.

However see the bridge girder in Fig.2, the cross-sections are different especially near the joint connecting the pillar and girder. This point was not considered and would be improved in the future.

7.4 Small deflection theory

The foregoing analysis is based on small deflection theory (Bao, 2008). It means the structural analysis is on the undeflected structure within the material mechanics assumption (Fan, 2004, Quimby, 2012). Superposition principle applies (Bao, 2008). It also means the beam curvature can be simplified as:

$$\frac{1}{\rho} = \frac{y''}{\left[1+(y')^2\right]^{3/2}} \approx y'' \tag{21}$$

This analysis is called linear analysis or first order analysis.

7.5 Geometrical non-linearity (Second order analysis)

If the structural analysis is on the deflected structure, it is geometrical non-linearity analysis and superposition principle no longer applies (Bao, 2008). This analysis can also be called as second order analysis (Bao, 2008, Chen and Sohal, 1995, Chen and Lui, 1991). One application of such theory is in deflection theory of long suspension bridge analysis (Bridge Science Research Institute, Major Bridge Engineering Bureau, the Ministry of Railways, 1996).

Buckling analysis is another kind of geometrical non-linearity (Second order analysis) (Bao, 2008). Euler formula adopts the forgoing Eq. (21). However, for large deflection theory, the beam curvature adopts the following definition: change rate of slope to arc length in the following equation (Tang, 1989).

$$\frac{1}{\rho}=\frac{y''}{\left[1+(y')^2\right]^{3/2}}=\frac{d\theta}{ds}\neq y'' \tag{22}$$

7.6 Buckling and Post-Buckling

Buckling analysis includes prediction of critical load of buckling, and determining the axial load-deflection relationship in post-buckling (Chen and Sohal, 1995, Chen and Lui, 1991, Tang, 1989). In post-buckling, the axial load exceeds the critical load required to induce buckling.

8. Conclusion

Because with the given external load, the internal load in bridge structure members are always not same. Thus it is rational to design a bridge with different cross-sections as to save the materials and decrease the self-weight of the structure. It means to safely and mainly economically design the bridges, the cross-sections of bridge girders or pillars are always variable. This engineering condition wants the FEM to include elements with different geometries. Although this problem is complex, this paper takes the first step and well solved the problem considering the distinction of the inertia area moment of pillars and girders in a FEM.

Influence lines of a rigid frame bridge could be determined with slope-deflection method not considering bar axial deformation (simplified finite element method) or slope-deflection method considering bar axial deformation (fully implemented finite element method). For the rigid frame bridge, there is only a little difference between Slope-deflection method ignoring beam element axial deformation (simplified FEM) and Slope-deflection method considering beam axial deformation (fully implemented FEM).

Influence line for moment at B from slope-deflection method coincides with the result from Müller-Breslau Principle.

Buckling and Post-Buckling of the rigid frame bridge could be done in the future.

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References

Bao, S. H. (2008). Structural mechanics. Wuhan : Wuhan University of Technology Press.

Bridge Mapper.(2011). Stoystown Road Overpass. http://www.bridgemapper.com/bridge_detail.php?ID=1717

Chen, W.F. & Lui, E. (1991). Stability design of steel frames. Florida: CRC Press.

Chen, W.F. & Sohal, I. (1995). Plastic design and second-order analysis of steel frames. New York : Springer-Verlag. http://dx.doi.org/10.1007/978-1-4613-8428-1

Fanous, F. (2000). Influence lines. http://www.public.iastate.edu/~fanous/ce332/influence/homepage.html.
Fan, Q. S. (2004). Engineering mechanics. Beijing: China Machine Press.
Grigorjeva, T., Juozapaitis, A., Kamaitis, Z., & Paeglitis, A. (2008). FINITE ELEMENT MODELLING FOR STATIC BEHAVIOUR ANALYSIS OF SUSPENSION BRIDGES WITH VARYING RIGIDITY OF MAIN CABLES. THE BALTIC JOURNAL OF ROAD AND BRIDGE ENGINEERING, 3(3), 121–128. http://dx.doi.org/10.3846/1822-427X.2008.3.121-128
Hibbeler, R. C. (1985). Structural Analysis. New York: Macmillan.
Heins, C. P. & Lawrie, R. A. (1999). Design of Modern Concrete Highway Bridges. New York: John Wiley and Sons.
Institute of Bridge Science of Bridge Engineering Department of Railways Ministry. (1996). Suspension Bridge. Beijing: Science and Technology Document Press.
Liu, Z. M. (1957). Reinforced concrete rigid frame bridge design. Beijing: Science and technology press.
Li, H. & Wu, L. H. (2005). Influence line construction of continuous curved beam bridge using singular-curved-beam method. Engineering mechanics, 22Sup., 26-30.
Li, Z. L. (2007). Bridge engineering. Wuhan: Huazhong University of Science & Technology Press Co., Ltd.
Leet, K. M., Uang, C. M., & Gilbert, A. (2008). Fundamentals of Structural Analysis. 3rd ed. McGraw-Hill Higher Education.
Niu, W. J. & Wang, Z. Y. (2011). Application of Flexibility Method to a Chain Suspension Stiffening Beam Bridge. Advanced Materials Research, 255-260, 1220-1224. http://dx.doi.org/10.4028/www.scientific.net/AMR.255-260.1220
Qi, L., Huang, F. L. & Jia, C. L. (2007). Alignment and stress control for construction of continuous rigid frame bridge. Journal of railway science and engineering, 4(2):29-33.
Quimby, T. B. (2012). A Beginner's Guide to Structural Engineering.http://www.bgstructuralengineering.com.
Shao, X.D. (2005). Bridge engineering. Beijing: China Communications Press.
Shan, J. (2011). Structural mechanics. Nanjing: Southeast University Press.
Tang, J. X., Wang S. T., & Pei, R.J. (1989). Theory of Structural Stability. Beijing:China Railway Publishing House.
Winter, G. & Nilson, A. H. (1979). Design of Concrete Structures. New York: McGraw-Hill Book Company.
Zhi. B. C. (1985). Structural mechanics. Beijing: the Central TV University Publishing House.