Numerical Simulation of Mixed Convection Heat and Mass Transfers with Film Evaporation of Water or Acetone in a Vertical Channel

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Abstract

A comparison numerical study of mixed convection heat and mass transfer with film evaporation in a vertical channel is developed. The emphasis is focused on the effects of phase change of two different liquid films having widely different properties on the heat and mass transfer rates in the channel. The induced laminar upward flow is a mixture of blowing air and vapour of water or acetone, assumed as ideal gases. A two-dimensional steady state and elliptical flow model, connected with variable thermo-physical properties, is used and the phase change problem is based on thin liquid film assumptions. The governing equations of the model are solved by FVM and the velocity-pressure fields are linked by SIMPLE algorithm. The numerical results, including the axial variations of friction factor, Nusselt numbers, Sherwood number and dimensionless film evaporation rate are presented for two values of inlet temperature and humidity. Within the range of inlet conditions under consideration, it was found that the vaporization of acetone film is considerable with the decrease of the inlet conditions. While increasing this inlet parameters leads to the condensation of water vapour on the wall and to better mass transfer rates related with acetone film evaporation. The dimensionless mass evaporating rate increases noticeably with the decrease of inlet temperature and the use of more volatile liquid film.

Keywords: Film evaporation; Mixed convection; Heat and mass transfer; Vertical channel

1. Introduction

Laminar mixed convection in vertical open channel flows with simultaneous heat and mass transfer walls have received considerable attention because they are important in many processes occurring in nature and engineering applications such as human transpiration, cooling of electronic equipment, refrigeration, air conditioning, desalination and many others.
Many studies were conducted which deal with different geometric configurations and various thermal and solutal boundary conditions. Lin et al. (1988) examined the combined buoyancy effects of thermal and mass diffusion on laminar forced convection heat transfer in a vertical tube. They showed that heat transfer in the flow is dominated by the transport of latent heat owing to the evaporation of the thin liquid film and that the ratio of the latent heat flux to the sensible heat flux has a minimum for a fixed wall temperature. Huang et al. (2005) and Jang et al. (2006) examined in detail the effects of the relative humidity of the moist air, wetted wall temperature, Reynolds number and aspect ratio of a vertical or an inclined rectangular duct with film evaporation along a porous wall. Evaporative cooling of a liquid film through interfacial heat and mass transfer in vertical ducts was investigated by Yan and Lin (1991), Yan (1998) and Feddaoui et al. (2001, 2006 and 2007). A liquid film streams along the plates with a temperature, which is higher than that of the downward airflow at the entrance. Their results showed that the influence of the evaporative latent heat transfer on the cooling of the liquid film depends largely on the inlet liquid film temperature and the inlet liquid mass flow rate. The major result drawn in theses papers is that the zero film thickness assumption which is adopted by many authors (Hammou et al., 2004; Azizi et al., 2007; Laaroussi et al., 2008; Kassim et al., 2011) is only valid for low liquid mass flow rates.

Hammou et al. (2004) studied numerically the effects of the inlet conditions on a downward laminar flow of humid air in a vertical channel with isothermal wetted walls. Cases of film evaporation and vapour condensation were considered. The effect of thermal and solutal buoyancy forces on flow characteristics is significant. Azizi et al. (2007) reconsidered the problem for both upward and downward mixed convection. According to the flow direction, the results show that in the case of relatively high temperature differences between ambient and isothermal wall flow reversal occurred for upward flows. Recently, Kassim et al. (2010) investigated the effect of inlet air humidity in a humidifier. Their results show that the performances of the humidifier are seriously affected when increasing the air humidity at the channel entrance as it induces condensation of the water vapour on the walls.

The above literature survey shows that the elliptic formulation has been systematically applied with the Boussinesq approximation and constant thermo-physical properties of humid air which are evaluated by the one-third rule. This assumption was investigated by Laaroussi et al. (2008) for simultaneous heat and mass transfer on laminar mixed convection in a vertical parallel-plate channel with film evaporation by considering two systems; air-water and air-hexane. They compared the Boussinesq and variable-density models at relatively high temperatures. In the most of papers mentioned above, the problem of the water evaporation was studied with the assumptions of constant properties and Boussinesq approximation. Whereas, other liquids play an important role in practical applications with large variations in thermo-physical properties. This scope motivates the present study. The main objective of this work is to examine the effects of the evaporation of two pure thin liquid films (water and acetone), selected for their large difference in properties, upon the heat and mass transfer rates in a vertical channel by analyzing the effect of inlet temperature and relative humidity of the air stream.

2. Problem description

The physical model adopted in this study is a humid air flowing upward inside a vertical parallel plate channel of height (L) and a width (H) with coordinate systems as shown in Fig 1. The fluid, with
constant properties corresponding to inlet temperature $T_0$ and humidity $\phi_0$, enters the vertical channel with uniform velocity $u_0$. The left plate of channel is wetted by a thin liquid film of water or acetone and isothermal, the right plate is insulated and dry.

For the mathematical formulation of the problem, the following simplifying assumptions are taking into consideration:

1. The flow is laminar, steady and two dimensional.
2. The binary mixtures of water or acetone vapor and air, is supposed to be an ideal mixture of perfect gas.
3. The gas-liquid interface is in thermodynamic equilibrium.
4. The liquid film is extremely thin (zero film thickness model) so it is regarded as a boundary conditions for heat and mass transfer.
5. Radiation heat transfer, viscous dissipations and other secondary effects are negligible.

![Fig. 1. Schematic diagram of the channel and coordinate system.](image)

2.1 Governing equations

With the above assumptions, the steady mixed convection heat and mass transfer in vertical channel can be described by the following governing equations.

\[
\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0 \tag{1}
\]

\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) - \rho g \tag{2}
\]

\[
\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left( \mu \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial v}{\partial y} \right) \tag{3}
\]

\[
\rho C_p u \frac{\partial T}{\partial x} + \rho C_v v \frac{\partial T}{\partial y} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) \tag{4}
\]

\[
\rho u \frac{\partial W}{\partial x} + \rho v \frac{\partial W}{\partial y} = \frac{\partial}{\partial x} \left( \rho D \frac{\partial W}{\partial x} \right) + \frac{\partial}{\partial y} \left( \rho D \frac{\partial W}{\partial y} \right) \tag{5}
\]
2.2 Boundary conditions:
At the channel inlet \((x = 0, 0 < y < H)\):
\[ u = u_a, \ T = T_u, \ W = W_o \]  \hspace{1cm} (6)
According to the Dalton’s law and the equation of state for ideal gas mixture, the inlet mass fraction of water vapour is calculated by the following expression:
\[ W_o = \frac{p_v(T_o)M_v}{p_v(T_o)M_v + (p_m - p_v(T_o))M_a} \]  \hspace{1cm} (7)
where \(M_v\) and \(M_a\) are the molecular weight of liquid vapour and air respectively. \(p_m\) is the thermodynamic pressure of gas mixture and \(p_v(T_o)\) is defined as:
\[ p_v(T_o) = \phi_0 p_v^{atm}(T_o) \]  \hspace{1cm} (8)
where \(\phi_0\) is defined as relative humidity.

At the left plate \((y = 0, 0 < x < L)\):
\[ u = 0, \ v = v_a, \ T = T_u, \ W = W_v \]  \hspace{1cm} (9)
Where the transverse velocity at the interface is given by:
\[ v_a = -\frac{D}{1 - W_v} \frac{\partial W}{\partial y} \bigg|_{y=0} \]  \hspace{1cm} (10)

At the right plate \((y = H, 0 < x < L)\):
\[ u = v = 0, \ \frac{\partial T}{\partial y} \bigg|_{y=H} = \frac{\partial W}{\partial y} \bigg|_{y=H} = 0 \]  \hspace{1cm} (11)

At the outlet of the channel \((x = L, 0 < y < H)\):
\[ \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial T}{\partial x} = \frac{\partial W}{\partial x} = 0 \]  \hspace{1cm} (12)
One of the constraints to be satisfied is the overall mass balance for the two systems of mixture, defined as:
\[ \int_0^H \rho u \, dy = \rho_s u_s H + \int_0^L \rho v \, dx \]  \hspace{1cm} (13)

2.3 Heat and mass transfer parameters.
Energy transport between the wetted wall and the air in the channel in the presence of mass transfer depends firstly on the fluid temperature gradient at the wetted wall, resulting in a sensible heat transfer and secondly on the rate of mass transfer, resulting in a latent heat transfer.

The local Nusselt number along the wall is defined as:
\[ Nu = \frac{h d}{k} = \frac{q_s d}{k(T_u - T_s)} = Nu_s + Nu_l \]  \hspace{1cm} (14)
Where \(h\) denote the local heat transfer coefficient, \(Nu_s\) and \(Nu_l\) are the local Nusselt numbers for sensible and latent heat transfer, respectively, and evaluated by:
\[ Nu_s = -\frac{2 H}{T_u - T_s} \left( \frac{\partial T}{\partial y} \right)_{y=0} \]  \hspace{1cm} (15a)
In a similar manner, the local Sherwood number on the wetted wall is given by

\[
Sh = \frac{h_w d_h}{D} = -\frac{2H}{(I-W_u)(W_u-W_w)} \left( \frac{\partial W}{\partial y} \right)_{y=0}
\]  

The fanning friction factor is:

\[
C_f = \frac{\mu}{(\rho u_y^2/2)}
\]  

In the present study, the thermo-physical properties of the gas mixtures are taken to be variable depending on temperature and concentration, they are calculated from the pure component data, Perry (1999), by means of mixing rules (Reid (1977)), applicable to any multi-component mixtures.

3. Numerical method

The system of coupled, non linear, elliptic partial differential eqs. (1)-(5), subject to their boundary conditions, has been solved numerically by using finite volume method. The velocity and pressure fields are linked by the Semi-Implicit Method for Pressure Linked Equations (SIMPLE) algorithm proposed by Patankar (1980). In order to get a mathematically well posed SIMPLE algorithm, overall continuity must be satisfied at outlet boundary, when computing the velocity field. The outlet plane velocities with the continuity correction are given by Versteeg and Malalasekera (1995):

\[
u_{sl,j} = \frac{u_{sl-1,j} \left[ \rho u_y H + \int_0^L \rho \nu_y \, dx \right]}{\int_0^L \rho u \, dy}
\]  

The resulting set of discretization equations can be cast into a tri-diagonal matrix equation and solved iteratively using the TDMA algorithm. The criterion of convergence of the numerical solution is based on the absolute normalized residuals of the equations that were summed for all cells in the computational domain. Convergence was considered as being achieved when the largest residual of all variables falls below \(10^{-6}\) at all grid points.

3.1 Grid independence

The physical domain was discretized into a structured grid, using the algebraic method for grid generation. A non-uniform grid system was used, with greater node density near the inlet and the walls where the gradients are expected to be more significant. The grid refinement tests were carried out to ensure the independence of the calculations on several grid sizes for the specific case of air-water system with \(Re = 300\), \(T_w = 20^\circ C\), \(T_0 = 30^\circ C\), \(\phi_0 = 10\%\) and \(\gamma = 1/100\), where \(\gamma\) is the aspect ratio defined by \(\gamma = H/L\). Table 1 presents the results of the local friction factor, sensible Nusselt and Sherwood numbers for each of these grid systems. It is clear that changes in \(Nu_u\), \(Sh\) and \(Cf\) with respect to the grid refinement are less than 3.5\%. Therefore, to optimize CPU resources with an acceptable level of accuracy, all parametric runs were made with the \((200 \times 50)\) grid.
Table 1 Comparison of local friction factor, Nusselt and Sherwood numbers for various grids

<table>
<thead>
<tr>
<th>x (m)</th>
<th>0.02</th>
<th>0.5</th>
<th>1.49</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid(x,y)</td>
<td>C_f</td>
<td>Nu_x</td>
<td>Sh</td>
</tr>
<tr>
<td>100x30</td>
<td>0.132</td>
<td>9.256</td>
<td>8.953</td>
</tr>
<tr>
<td>150x40</td>
<td>0.130</td>
<td>9.118</td>
<td>8.824</td>
</tr>
<tr>
<td>200x50</td>
<td>0.129</td>
<td>9.050</td>
<td>8.757</td>
</tr>
<tr>
<td>250x60</td>
<td>0.131</td>
<td>9.343</td>
<td>8.712</td>
</tr>
<tr>
<td>300x70</td>
<td>0.128</td>
<td>8.972</td>
<td>8.943</td>
</tr>
</tbody>
</table>

3.2 Code validation
The developed code based on the mathematical model above is validated in two ways by reproducing solutions for some results in the literature.

3.2.1 Laminar mixed convection with thermal diffusion.
The code is validated with two different cases: First, a combined forced and laminar mixed convection problem using the Boussinesq approximation is chosen, a vertical parallel plate channel of aspect ratio $\gamma = 1/50$ with a constant and uniform temperature at the boundaries is selected $T_w = 10^\circ C$ and $T_c = 60^\circ C$. The working fluid is air, which enters at the bottom of the channel with uniform velocity profile, $Re = 300$, the Grashof number is varied ($4.71 \times 10^4 \leq Gr \leq 1.27 \times 10^5$) by varying the width of the channel ($0.02 m \leq H \leq 0.06 m$). Results of the local Nusselt number ($Nu$) along the non-dimensional axial length ($x/d_Pe$) are compared with results of Desrayaud and Lauriat (2009). As shown from Fig 2, a good agreement is observed. The discrepancies are less than 2%.

![Fig. 2. Validation of calculated Nusselt number at the wall.](image)

3.2.2 Laminar mixed convection with solutal diffusion.
The comparison is done for a vertical channel of aspect ratio $\gamma = 1/100$ with constant and equal temperature at the boundaries ($T_w = T_c = 327.5 K$), ($Gr = 0$). Dry air entered the channel with uniform downward velocity and $Re = 300$, the vertical parallel plates are wetted with water ($W_v = 0.1 kg_{vapour}/kg_{air}$). Fig 3 shows a comparison of velocity profiles calculated by Laaroussi et al.
(2008) and our results using the Boussinesq approximation, which are in excellent agreement, where the largest velocity difference is about 2.5%.

![Graph of vertical velocity profiles for Re = 300, W = 0.1 kg kg⁻¹](image)

Fig. 3. Comparison of vertical velocity profiles for Re = 300, W = 0.1 kg kg⁻¹

Through these program tests and successful comparisons, we conclude that the model and the present numerical code are considered to be suitable for the present investigation.

4. Results and discussion

This study will examine the effects of the evaporation of two different thin liquid films, which exhibits different volatilities, on the heat and mass transfer rates. The induced flow is a mixture of the blowing dry air and vapour of the liquid film. Heat and mass transfer for the mixed convection depends on the dimensionless parameters Gr, Gr, Pr and Sc which are interdependent for a given mixture, and cannot be arbitrarily assigned. Instead, the temperature of the dry air and Reynolds number at the inlet section are introduced as the independent variables for water and acetone thin film evaporation. The numerical simulations assume the following conditions: Re = 300, γ = 1/100 and T = 20°C. The mass fraction at the left plate of channel is W = 14.44 g kg⁻¹ for water and W = 38.14 g Kg for acetone. The inlet temperature was assigned T = 30°C and 100°C and two values of the inlet relative humidity φ = 10% and φ = 70%. The thermo-physical properties and operating conditions used for the simulation and other relevant parameters are presented in tables 2 and 3, respectively.

<table>
<thead>
<tr>
<th>Liquid film</th>
<th>M (g/mol)</th>
<th>T (°C)</th>
<th>P (KPa)</th>
<th>ρ (Kg/m)</th>
<th>μ (N/m/s)</th>
<th>h (Kj/Kg)</th>
<th>D (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>18.015</td>
<td>100</td>
<td>2.137</td>
<td>1.196</td>
<td>1.792×10⁻⁵</td>
<td>2432.7</td>
<td>2.518×10⁻⁵</td>
</tr>
<tr>
<td>Acetone</td>
<td>58.08</td>
<td>56.5</td>
<td>24.50</td>
<td>1.480</td>
<td>1.330×10⁻⁵</td>
<td>561.2</td>
<td>1.061×10⁻⁵</td>
</tr>
</tbody>
</table>
Table 3 Values of parameters for cases under study

<table>
<thead>
<tr>
<th>system</th>
<th>T₀ (°C)</th>
<th>Ø₀ (%)</th>
<th>W₀ (g/Kg)</th>
<th>βₘ</th>
<th>Grₜ</th>
<th>Grₘ</th>
<th>N</th>
<th>Pr</th>
<th>Sc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air-Water (Sys 1)</td>
<td>30</td>
<td>10</td>
<td>2.606</td>
<td>0.604</td>
<td>-35908</td>
<td>-131589</td>
<td>0.140</td>
<td>0.670</td>
<td>0.607</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>70</td>
<td>18.42</td>
<td>0.602</td>
<td>-35572</td>
<td>-18364</td>
<td>0.072</td>
<td>0.707</td>
<td>0.610</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>10</td>
<td>64.614</td>
<td>0.597</td>
<td>-131589</td>
<td>-18364</td>
<td>0.140</td>
<td>0.702</td>
<td>0.703</td>
</tr>
<tr>
<td>Air-Acetone (Sys 2)</td>
<td>30</td>
<td>10</td>
<td>8.07</td>
<td>-0.552</td>
<td>-53646</td>
<td>-320886</td>
<td>5.564</td>
<td>0.695</td>
<td>1.137</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>70</td>
<td>55.19</td>
<td>-0.558</td>
<td>-57668</td>
<td>-320886</td>
<td>5.564</td>
<td>0.695</td>
<td>1.137</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>10</td>
<td>177.00</td>
<td>-0.570</td>
<td>-263462</td>
<td>-145414</td>
<td>0.552</td>
<td>0.700</td>
<td>1.188</td>
</tr>
</tbody>
</table>

The thin liquid films properties shown in table 2 indicate the notable differences between the two fluids, particularly; acetone has the lowest latent heat of vaporization. On the other hand, acetone is the most volatile (lower boiling temperature, \( T_b = 56.5°C \)) and heavier than the water film.

The magnitudes of the buoyancy forces for each system can be parameterized through thermal and solutal Grashof numbers \( Gr_t \) and \( Gr_u \). For both systems with \( T_0 = 30°C \) and \( \phi_0 = 10\% \) the case of forced convection \( Gr_t = Gr_u = 0 \) is also reported for comparison purposes. Since the temperature of wall is less than the inlet temperature \( (T_w < T_0) \), it is noticed for all cases listed in table 3, the thermal Grashof are negative, thus the corresponding buoyancy acts in the opposite direction of the air flow. The solutal Grashof number may be positive or negative; its sign depends on the solutal coefficient of volumetric expansion and inlet humidity. The relative importance of thermal and solutal buoyancy forces is denoted by the buoyancy ratio \( N \), and is defined as the ratio between the solutal and thermal buoyancy forces.

\[
N = \frac{Gr_u}{Gr_t} = \frac{\beta_u (W_u - W_0)}{\beta_t (T_u - T_0)}
\]

\( \beta_u \) is evaluated at the reference mass fraction calculated by using the one-third rule, Laaroussi et al. (2008). The air-water system is characterized by two buoyancy ratio, the first \( (-1 < N < 0) \) corresponding to the thermal-dominated opposing flow and the second ratio \( (0 < N < 1) \) indicating that the induced flow is influenced by thermal buoyancy which acts in the flow direction. For the case of air-acetone system the buoyancy ratio is larger than unity for \( T_0 = 30°C \) and for relative humidity \( \phi_0 = 10\% \) and \( 70\% \) thus the solutal buoyancy forces are prevalent and aiding flow.

4.1 Streamlines and isotherms
Streamlines patterns obtained from numerical simulation for the effect of inlet temperature are presented in Figs. 4 and 5. These figures provide a better insight into the nature of the flow field. For the air-water system, the flow is upward at every location (Fig 4a). At the wetted wall \( (y = 0) \) and near the entrance, the streamlines are slightly narrowed showing an acceleration of the fluid in this region. The cells located at the core of the inlet section (Fig 4a) correspond to the maximum of the axial velocity owing to the aiding buoyancy forces. Towards the channel exit, the streamlines are nearly parallel, indicating that the flow is developed.
Fig. 4b illustrates the effect of increasing the inlet temperature \( T_0 \), showing that near the entrance and close to the walls the streamlines are deflected. The cells centered close to the wetted wall correspond to maximal upward velocities induced by the aiding buoyancy forces. While the cell attached to adiabatic wall \( y = H \) having a zero value determines the boundary of the region where the axial velocity is downward (negative value).

For the air-acetone system, Fig. 5 shows the isovalues of axial velocity, which exhibits features more pronounced than those observed in Fig. 4b. In comparison with the air-water system, the main changes in the velocity profiles are observed near the walls and inlet section: The center of the closed isolines located near the wetted wall shows the locus of the point where axial velocity is maximum and more pronounced for a higher inlet temperature \( T_0 \). This is a direct consequence of the aiding buoyancy forces, especially large solutal buoyancy effect. As can be clearly seen from the streamlines, the closed isolines adjacent to the insulated wall reveal the onset of flow recirculation where the axial velocity is downward. Also, it is observed (Fig. 5a and 5b) that the solutal buoyancy forces produce larger extents of the regions with maximum velocity adjacent to the wetted wall and of the recirculation cells at the adiabatic wall. Therefore the maximum velocity isolines reach the outlet of the channel as the gas mixture proceeds downstream revealing that the flow is not developed.

The thermal fields presented in Figs. 6 and 7 correspond to the streamline plots shown in Figs 4 and 5. By comparing the isotherms for the two flow systems under the same inlet conditions, we noticed that as the flow goes downstream the air-acetone system is more cooled than the air-water. In the case of air-acetone system, the effects of reversal flow at the adiabatic wall are clearly seen in Figs 7 comparing to Figs 6. Also, it can be pointed out from these figures that the air-water flow is hotter as it approaches the adiabatic wall than air-acetone flow, particularly, with the increase of inlet temperature \( T_0 \).

![Fig. 4. Streamlines for air-water system.](image)

![Fig. 5. Streamlines for air-acetone system.](image)
4.2 Friction factor

In this section, we will consider the comparison of friction factor profiles for the two flows systems, because of their usefulness in clarification of evaporation phenomenon between the wetted wall and the insulated one.

The axial distributions of friction factor $C_f$ of the two systems are illustrated in Figs. 8a and 8b. The figures show the effects of $T_o$ and $\phi_o$ on the film evaporation. It is clearly seen from both figures that $C_f$ increases with the increase of inlet temperature $T_o$. Besides, the increase of $\phi_o$ leads to a slight decrease in the first one-third of the channel at the inlet of $C_f$ for the air-water system as shown in Fig 8a. In other hand, for the air-acetone system (Fig 8b), $C_f$ decreases exhibiting a different behaviour than the first system. This is due to the fact that the buoyancy forces mostly of solutal origin decelerate the flow near the wetted wall and consequently $C_f$ is decreased. The case of forced convection is also reported to show the effects of buoyancy forces. As seen from the Figs 8a and 8b, the axial evolution of friction factor $C_f$ is monotonic in the case of forced convection and tends towards an asymptotic value at the channel exit. Indeed, in this case, $C_f$ presents local maxima located near the channel entrance owing to the maximum of axial velocity as seen in Figs.4 and 5. These maxima are more pronounced as $T_o$ is increased, which in turn, causes the larger aiding buoyancy as the flow moves downstream. The velocity near the wall increases and consequently $C_f$ is increased. By comparing the two Figs (8a and 8b), it is also depicted that the friction factor of air-acetone system is always larger than the air-water system because of prevalent of solutal buoyancy forces compared to those of air-water system. For the case of air-acetone system (Fig 8b), after a certain axial locations the axial friction factor begins to decrease monotonically and merge at the exit with the increase of $T_o$ showing that the induced flow did not reach a fully developed state because the friction factor didn’t ceased to decrease as a function of distance by comparing with forced convection. For the case of air-water system (Fig 8a), as the induced flow moves downstream $C_f$ decreases towards the same asymptotic value of
\( C_f = 0.082 \approx 24/Re \) at the channel outlet which corresponds to that for fully developed flow in forced convection, Bejan (2004).

![Graph of friction factor](image)

**Fig. 8.** Axial evolution of the friction factor: (a) air-water system, (b) air-acetone system

### 4.3 Axial evolution of the sensible Nusselt and Sherwood numbers

Shown in Figs 9a and 9b are the distributions of the local sensible Nusselt number (\( Nu_s \)), for the two flow systems respectively, along the wetted wall. These figures follow the same trend seen earlier in the above section. At the channel entrance large \( Nu_s \) is noted for a higher \( T_0 \) for both systems. Nusselt number \( Nu_s \) increases slightly when increasing \( \phi_0 \) to 70\% for the cases of air-water system, Fig 9a. As the induced flow moves downstream, \( Nu_s \) decreases monotonically towards the same asymptotic value at the channel outlet which is equal to the analytical value (\( Nu_s \approx 4.86 \)) for fully developed flow in forced convection, Bejan (2004). In this limiting case (\( Ri = 0 \)), the buoyancy force is uniformly distributed across the channel. This behaviour is absent in the case of air-acetone flow, for which \( Nu_s \) exhibits local maximum that corresponds to the acceleration of flow close to the wetted wall. The latter is confirmed by the streamlines of air-acetone system as shown from Fig 5.

Fig 9b shows that the \( Nu_s \) number increase with the increase of inlet \( T_0 \), this is due the slowing down of the flow when the \( Gr_s \) number is increased causing the reversal flow which accelerates the fluid flow along the humid wall and thus increases the heat transfer efficiency. As the flow moves downstream the \( Nu_s \)-plots of air-acetone flow decreases and merge at the exit with the increase of \( T_0 \).
To characterize the mass transfer, the variations of local Sherwood number along the wetted wall are depicted in Figs 10a and 10b for the two systems. It is clear that the results are similar in trend to those of the sensible Nusselt number. By comparing $Sh$ and $Nu$, curves of the two flow systems, we found that the distributions of Sherwood number in Fig 10a resemble those of $Nu$, in Fig 9a for the case of air-water system. This is because Prandtl number $Pr$ and Schmidt number $Sc$ are the same order of magnitude (see table 3). For the air-acetone system, the magnitude of $Sh$ in Fig 10b is greater with variation of both $T_0$ and $\phi_0$ than that of $Nu$, number as shown from Fig 9b. This is owing to the fact that, for the air-acetone system, the Schmidt number is greater than Prandtl number (table 3). In other words, this is due to the large concentration gradient at the wall and the lower mass diffusion coefficient. However, Figs 10a and 10b show that $Sh$ number are not influenced by increasing the inlet concentration.

4.4 Axial evolution of the latent Nusselt number
According to the results in Figs 11a and 11b, the differences in latent Nusselt number plots ($Nu_l$) are considerably higher. As expected, increasing the inlet temperature leads to the increasing of the latent
Nusselt number with associated changes in thermo-physical properties. Although the effects of relative humidity on the sensible Nusselt and Sherwood numbers are not significant, their effects on the latent heat transfer are considerable.

For the case of air-water system the profiles of $Nu_L$ can be positive or negative as shown in Fig 11a, it is found that the two $Nu_L$-plots corresponding to $T_o = 100°C$ and $\phi_0 = 70%$ are positive, but for the system $T_o = 30°C$ and $\phi_0 = 10%$ for both mixed or forced convection the $Nu_L$ profiles are negative. On the other hand the $Nu_L$ is positive as seen in Eq.(15a), Thus the direction of latent heat exchange can be directed or opposed to that of sensible heat exchange. Therefore, the energy is transported from the wetted wall to gas stream through condensation or inversely through evaporation.

In the case of air-acetone system (Fig 11b.), and referring to the expression of latent Nusselt number eq (15b), $Nu_L$ profiles are negative because of the wall temperature is less than the average one ($T_w < T_o$) not shown here for the sake of brevity, also comparing the ordinate scales of the air-acetone system points out that the magnitude of $Nu_L$ is relatively very larger than that of sensible Nusselt number $Nu_s$ over the entire channel length (Fig 10b.), therefore the heat transfer due to the transport of latent heat is predominant than that due to the transport of the sensible heat in the wall. Additionally, by comparing the air-acetone system to the air-water flow, the result is that under the same inlet conditions, the heat exchange during the evaporation of the thin liquid film of acetone is much effective than that with water film evaporation. Thus the cooling of the induced flows is enhanced with the decreasing of $T_o$ and $\phi_0$ because acetone film has large latent Nusselt number (Fig 11b.).

4.5 Axial evolution of the net mass flow rate
As mentioned above, interfacial heat and mass transfer in the wetted wall system is dominated by film vaporisation. Therefore, the amount of vapour added to the gas stream due to film vaporisation is important for a more volatile component in improving our understanding of the heat and mass transfer rates. To quantify the film evaporation a nondimensional accumulated mass evaporation rate, $Mr$ is introduced,
The effect of inlet temperature $T_o$ and relative humidity on the distributions of dimensionless accumulated mass evaporation rate ($M_r$) for the two flow systems and for various cases are illustrated in Figs 12a and 12b. By comparing $M_r$-curves, the differences are rather substantial. It was found that under the same inlet conditions, the occurrence of condensation on the wetted wall is observed for the case of film water (Fig 12a). This is explained by the decrease of $M_r$ with $x$ when increasing the inlet humidity to $\phi_o = 70\%$ and more pronounced when increasing the inlet temperature to $T_o = 100^\circ C$. It is noted that for the case of $T_o = 30^\circ C$ and $\phi_o = 10\%$ for both mixed and forced convection, the evolution of $M_r$ is similar. As for the case of air-acetone system, as shown in Fig 12b, the acetone vapour added to the gas stream due to film evaporation is more significant. The largest $M_r$ is found for a system with smaller $T_o$ and $\phi_o$ are respectively for acetone and water films $57\%$ and $1.1\%$.

$$M_r = \frac{1}{\rho} \rho u \int_0^L \rho u H \, dx$$  \hspace{2cm} (20)

![Fig. 12. Distributions of accumulated evaporation rate: (a) air-water system (b) air-acetone system](image)

5. Conclusion

This paper presents a numerical simulation of the evaporation by laminar mixed convection in a vertical channel formed by two parallel plates; one is isothermal and wetted by a thin liquid film of water or acetone, the second plate is insulated. Specifically, we have investigated the effects of inlet air temperature and humidity on the heat and mass transfer rates. Under the same inlet conditions, the major comparative results are briefly summarized for both water and acetone film evaporations as follows:

1. Friction factor is significantly influenced by buoyancy forces compared with the corresponding results of forced convection. A higher $T_o$ results in increasing $C_f$ on the wetted wall which is more pronounced with the phase change of a component heavier than the carrier gas.
2. Increasing of inlet temperature leads to high sensible heat transfer and large mass transfer rate for air-acetone system followed by air-water system. Thus the better mass transfer rates related with film evaporation are found for a system with low mass diffusion coefficient.
3. Within the range of inlet conditions under consideration, larger $N_{Lu}$ in connection with evaporation is noted for the acetone film with lower $T_o$ and $\phi_o$, while condensation takes place on wetted wall for the water film with higher $T_o$ and $\phi_o$.

4. The dimensionless mass evaporating rate increases noticeably with decreasing $T_o$ and $\phi_o$ and the use of more volatile thin liquid film.

References


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