A Study on Critical Radius and Crossover Radius of Insulation for Various Heat Transfer Problems

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Abstract

Insulating the radial thermal system is based on the concept of critical radius which is widely discussed in the literature. When it is required to increase heat dissipation as in electrical transmission, critical radius can be used for maximum heat dissipation. But if it is desirable to decrease heat dissipation, critical radius alone cannot be sufficient. For such systems, the concept of crossover radius has to be used and is defined as the radius greater than the critical radius such that the heat transfer with the corresponding amount of insulating material is equal to that of the bare thermal system. In this paper an investigation of various heat transfer parameters like thermal conductivity, Convective heat transfer coefficient, length of cylinder on the effect of crossover radius of cylindrical insulating system is carried out using a computer code developed on 'C' platform. The code is validated by comparing critical radius profiles for thermal insulating system with those reported in literature. From the obtained results, novel conclusion for the design of radial thermal insulating systems (Cylinder) can be drawn.

Keywords: Crossover radius; Critical radius; Heat transfer rate; Thermal conductivity; Convective heat transfer coefficient

1. Introduction

Insulating the radial thermal system is used in many fields such as in industries, electrical transmission, refrigeration, cryogenics etc. In certain cases it is desirable to increase heat dissipation (cable wires etc.) and in some cases it is desirable to decrease heat dissipation (steam pipes, air conditioner ducts etc.). The application of critical radius for radial thermal systems is widely reported in the literature. When it is desirable to increase heat dissipation as in electrical transmission, critical radius can be used as radius of insulation for maximum heat dissipation. However, when it is desirable to decrease heat dissipation, critical radius is not always sufficient which is obvious from Fig 1.

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For such thermal systems the concept of crossover radius comes into play, and it is defined as the radius greater than the critical radius such that the heat transfer with the corresponding amount of insulating material is equal to that of the bare thermal system. Inspite of the above facts, only a very few studies have been reported in the literature on the crossover radius. Richard et al. (2007) developed an explicit relation of crossover radius on an insulated cylinder with forced convective heat transfer. He used the same assumptions as that used for deriving the relation for critical radius of insulation and his study was focused only on cylindrical system. Sunan et al. (2013) investigated the crossover radius of a transient cylindrical system so as to provide the necessary design criteria of a cylindrical system subjected to transient conditions. E.M.Sparrow et al. (1970) analyzed the critical radius problem of a 2-D horizontal cylinder subjected to natural convection with air as a fluid. He used the correlation of Mc Adams, Churchill & Chu, and Morgan and found that the Morgan correlation gives good agreement between 1-D and 2-D cylindrical systems. N.B.Totala et al. (2013) analyzed the critical radius of insulation problem of a cylindrical system in which he reported the variation in heat loss against critical radius experimentally. And while plotting the graph for heat transfer v/s outer radius he found the crossover point, but not emphasized much on crossover radius. M.R. Kulkarni (2004) showed that the concept of critical radius is not sufficient when it is required to minimize heat gain and heat loss. And he used the crossover radius to address design issues of such insulated system. The study made by Kulkarni mainly focused on cylindrical and spherical system with Biot Number ranging from (1< Bi <2).

From the above studies, it is obvious that except for Kulkarni (2004), none of the investigators have paid attention towards the effect of parameters on the heat transfer process of cylindrical system. Though Kulkarni (2004) had made an attempt, it was only confined to the Biot number criteria. Thus the prime objective of this investigation is to analyze the effect of various heat transfer parameters like thermal conductivity (k), heat transfer coefficient (h), and length (L) on a cylindrical insulated system.

![Fig.1. Critical & Crossover radius of insulation](image-url)
2. Analysis and Methodology

Consider a cylindrical system of radius ‘r_0’ which is insulated with a material of conductivity ‘k’. Let the length of the cylindrical system be ‘L’. The radius of the system after insulation becomes ‘r_0’. The inner temperature of insulation is ‘T_i’ and the outer surface temperature exposed to a convection environment is ‘T_∞’. By assuming a constant convective heat transfer coefficient ‘h’ and by neglecting any contact resistance and radiation, then using thermal network the heat transfer rate can be derived as (2011, 1983):

\[
Q = \frac{(T_i - T_∞)}{\frac{\ln{\frac{r_o}{r_i}}}{2\pi Lk} + \frac{1}{2\pi r_o h}}
\]

From above relation it is obvious that heat transfer rate is dependent on thermal conductivity ‘k’ and heat transfer coefficient ‘h’ and length ‘L’ of the cylinder. In order to study the effect of these parameters, a simple computer code is written on ‘C’ platform with above varying parameters as inputs. The heat transfer results obtained is post processed via excel plot and presented.

3. Validation

The validity of final code is ascertained by comparing critical radius profiles for thermal insulating system with those reported in Literature (2011, 1996 and 1983). The value of critical radius for cylindrical system is k/h. One such comparison for a heat transfer process of cylindrical system using parameters like thermal conductivity k=0.085 W/mK, heat transfer coefficient h=10 W/m²K, inlet temperature T_i=800K, outlet temperature T_∞=300K, inner radius r_i=0.0015m, & length=20x10⁻²m is shown in the figure 3 below.

It is very obvious from the below graph that critical radius very well matches with theoretical results i.e. r_c=k/h, i.e. 0.0085m.
4. Results and Discussion

This section describes the effect of thermal conductivity \( k \), heat transfer coefficient \( h \) and length \( L \) on the crossover radius. The inlet temperature \( T_i \), outlet temperature \( T_\infty \), Inner radius \( r_i \) are taken to be 800K, 300K & 0.0015m respectively. The range of parameters selected for study is as follows:

- Thermal conductivity \( k = 0.065 \text{W/mK} \) to \( k = 0.08 \text{W/mK} \).
- Heat transfer coefficient \( h = 10 \text{W/m}^2\text{K} \) to \( h = 13 \text{W/m}^2\text{K} \).
- Length = 20x10^{-2}m to 50x10^{-2}m.

4.1 Effect of thermal conductivity \( k \) on crossover radius

Figure 4 depicts the effect of thermal conductivity on crossover radius. It is evident that as thermal conductivity increases, the crossover radius increases which is more evident in figure 5. As expected, it is also obvious that as the thermal conductivity of insulating material decreases, the heat transfer rate also decreases. It can also be observed that with increase in thermal conductivity, the rate of increase in heat transfer decreases. And a closer view of graph indicates that with increase in thermal conductivity the critical radius drift to right side which shows that for high thermal conducting insulating material, the radius of insulation required for heat dissipation is also high. These factors are the prime factors for the design of thermal systems operating at high temperatures.
4.2 Effect of convective heat transfer coefficient on crossover radius

From figure 6 it is very much clear that, as the value of heat transfer coefficient increases the crossover radius decreases, which is also shown in figure 7. It is also observed that as heat transfer coefficient increases, the rate of increase of heat transfer decreases and critical radius shift towards left side. This indicates that for constant thermal conductivity, with increase in amount of heat transfer coefficient, the value of critical radius decreases. So it can be concluded that for the thermal systems operating on a low heat transfer coefficient side such as the gas side, the crossover radius requirement to minimize the heat transfer rate is large. These are also some of the important factors which the designer has to keep while designing the thermal systems operating at high temperatures.
4.3 Effect of length of pipe on crossover radius

Figure 8 depicts that with increase in the length, the crossover radius also increases. This is also shown in figure 9. It is observed that with increase in length, the critical radius remains same which is in concurrence with the literature. It is also observed that with increase in length, the rate of increase in heat transfer remains nearly constant. These factors are very helpful for the design of radial thermal systems operating at high temperatures.
5. Conclusion

Insulating the radial thermal system based on the concept of critical radius for reducing heat transfer rate serves as only a necessary condition but not enough. Therefore, to address the design issues of various thermal systems, the concept of crossover radius is very much necessary. It is defined as the radius greater than the critical radius such that the heat transfer with the corresponding amount of insulating material is equal to that of the bare thermal system. In this paper, an analytical study on the affect of different heat transfer parameters on the crossover radius is conducted and analyzed. It is found that for increasing values of thermal conductivity of the
insulating material and length of the cylinder, crossover radius increases. However, on increasing the value of convective heat transfer coefficient the value of crossover radius decreases. It is also observed that as the thermal conductivity and heat transfer coefficient increases the rate of increase in heat transfer decreases, while for increase in length, the rate of decrease of heat transfer is nearly constant. These factors serve as the prime factors for the design of radial thermal systems (both for cylindrical and spherical insulation systems).

References