Investigations of Natural Convection Heat Transfer in Nanofluids Filled Horizontal Semicircular-Annulus using Nonhomogeneous Dynamic Model

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Abstract

In this paper, the problem of unsteady convective flow of nanofluid in a horizontal semicircular-annulus using nonhomogeneous dynamic mathematical model has been investigated. The outer wall of the annulus is considered a colder wall and the inner is maintained at three different temperatures (constant, quadratic and sinusoidal) while two other walls are thermally insulated. The Galerkin weighted residual finite element method has been employed to solve the governing partial differential equations after converting them into non-dimensional form using suitable transformation of variables. In the numerical simulations, the cobalt-kerosene nanofluid has been taken to gain insight into the flow, thermal fields as well as concentration levels of nanofluids. Local Nusselt number and temperature gradient magnitude on the hotter and colder wall are displayed as line graphs. The average Nusselt number for cobalt-kerosene nanofluid is displayed as line graphs for different flow parameters including amount, shape and size of nanoparticles. Also, the average Nusselt number is presented as a bar diagram for 12 types of nanofluids. The result shows that 1-20 nm size nanoparticles are uniform and stable in the solution. The average Nusselt number increases significantly, as nanoparticle volume fraction, Rayleigh number increases and nanoparticle diameter decreases. Sinusoidal heating inner wall of annulus exhibits higher heat transfer rate. Average Nusselt number of cobalt-kerosene nanofluid is much higher than that of other 11 types of nanofluids which are studied in the present analysis.

Keywords: Nanofluids; Semicircular-annulus; Dynamic model; Heat transfer; Brownian diffusion; Thermophoresis

1. Introduction

Natural convection heat transfer in a horizontal annulus between concentric semicircles has been the focus of many investigations in recent ages because of the growing need for a better understanding of this phenomenon in technological and engineering applications such as electronic packaging, food...
process, cooling channels of nuclear reactors and solar collector-receiver. There are several researches on both numerical and experimental investigation of natural convection heat transfer between two concentric circular cylinders. Powe et al. (1971) studied numerical solution for natural convection in cylindrical annuli and classified the flow patterns conferring to suitable combinations of the Rayleigh number and the radius ratio. Kuehn and Goldstein (1976) conducted an experimental and theoretical study of natural convection in an annulus between horizontal concentric cylinders. They observed that for air, the flow started to oscillate near Rayleigh numbers of $10^5$ and steady laminar boundary layer regime exists between Rayleigh numbers of $3 \times 10^4$ and $10^5$. Onyegegbu (1986) studied the heat transfer within the annular gap of two infinitely long isothermal horizontal concentric cylinders using the Milne-Eddington approximation. He has showed that declining Planck number, increasing the degree of non-grayness of the fluid or increasing optical thickness, increases the total heat transfer and diminishes the induced buoyant flow intensity and velocities. Karki and Patankar (1989) presented a numerical study for laminar mixed convection in the entrance region of a horizontal annulus with a fixed radius. Their result showed that as Grashof number increases, Nusselt number and friction factor increases in both developing and fully developed regions. Ghaddar (1992) reported the numerical results of natural convection from a uniformly heated horizontal cylinder placed in a large air-filled rectangular enclosure and observed that flow and thermal behavior depend on heat fluxes impose on the inner cylinder within the isothermal enclosure. Nonino and Giudice (1996) conducted finite element analysis of laminar mixed convection in the entrance region of horizontal annular ducts with different radius ratios. Cesini et al. (1999) conducted the numerical and experimental analysis of natural convection from a horizontal cylinder enclosed in a rectangular cavity. Their result showed that the Nusselt number distribution on the upper cylinder surface is effectively influenced by the buoyancy plume originating from the lower cylinder. The effect of surface radiation on conjugate natural convection in a horizontal annulus driven by inner heat generating solid cylinder is investigated by Shaija and Narasimham (2009). Their result showed that surface radiation enhances the overall Nusselt number. Mohammed et al. (2010) investigated experimentally the force and free convection for thermally developing and fully developed laminar airflow inside horizontal concentric annuli. Their investigation revealed that the heat transfer rate is greater in the developing flow than the fully developed flow over a significant portion of the annuli. There are many numerical and experimental investigations on convection heat transfer of the concentric annulus using air and water as working fluids. The notable works are from Lunberg et al. (1963), Kuehn and Goldstein (1987), Lei and Trupp (1990), Moukalled and Acharya (1996), Char and Hsu (1998), Haldar (1998) and Yoo (1998).

By the emergence of nanoscience and nanotechnology, nanofluids have been developed to enhance the heat transfer rate instead of conventional heat transfer fluids like water and air. Nanofluids are produced using base fluids like water, ethylene glycol, engine oil, pump oil, glycerol, etc. and 1-100 nm size particles are made from various materials. Nanofluids have significantly higher thermal conductivity and heat transfer rate than conventional heat transfer fluids. Excellent result on thermal conductivity of suspensions by dispersing nanoparticles has been found by Lee et al. (1999), Das et al. (2003) and Xuan et al. (2003). Tremendous results on heat transfer enhancement of nanofluids have been published by Pak and Cho (1998), Xuan and Li (2003), Wen and Ding (2004), Yang et al. (2005) and Heris et al. (2006). A comprehensive review on nanofluid thermal conductivity as well as augmentation of heat transfer coefficient and its application can be found in Uddin et al. (2016) and Molana (2016). Different types of nanofluids are available commercially and used successfully for the enhancement of heat transfer. Among them CuO-water, Al$_2$O$_3$-water, ZnO-water, Cu-water,
CNT-water nanofluids are very common. Now a day, research on nanofluids containing Fe$_3$O$_4$, cobalt and nickel as nanoparticles is ongoing. These types of nanofluids showed promising characteristics to enhance heat transfer rate. Sheikhholeslami and Ganji (2014) studied ferrohydrodynamic and magnetohydrodynamic effects on ferrofluid flow in a semicircular annulus. They concluded that increment of solid volume fraction with the increments of Rayleigh number played positive role for enhancement of heat transfer.

The numerical investigations of natural convection heat transfer in different types of concentric annulus filled with nanofluids are the most recent attention by the different investigators. Abu-Nada et al. (2008) investigated natural convection heat transfer enhancement in horizontal concentric annuli filled with nanofluid. They found that at low Rayleigh number, nanofluids containing nanoparticles with higher thermal conductivity cause more enhancement in heat transfer. Hussain and Hussein (2010) investigated natural convection phenomena in a uniformly heated circular cylinder immersed in square enclosure filled with air at different vertical locations. Their result showed that small Rayleigh numbers does not have much influence on the flow field while at high Rayleigh numbers have considerable effect on the flow pattern. Zi-Tao et al. (2012) studied the transient natural convection heat transfer of aqueous nanofluids in a horizontal annulus between two coaxial cylinders. They found that as nanoparticle volume fraction is increased, the time-average Nusselt number is gradually lowered at constant Rayleigh number. Soleimani et al. (2012) investigated natural convection heat transfer in a nanofluid filled semi-annulus enclosure. Their result showed that there is an optimal angle of turn in which the average Nusselt number is maximum for each Rayleigh number. Yang et al. (2013) investigated convective heat transfer of nanofluids in a concentric annulus and found that Nusselt number has optimal bulk mean nanoparticle volume fraction for alumina water nanofluid, whereas it only increases monotonously with bulk mean nanoparticle volume fraction for titania-water nanofluid. There are more tremendous effort has been made to analyze the concentric annulus using nanofluids. The notable numerical investigations of natural convection heat transfer on concentric annulus using nanofluids include Arefmanesh et al. (2012), Sheikhholeslami et al. (2014), Seyyedi et al. (2015) and Bezi et al. (2015).

Our aim is to find out the enhancement of heat transfer rate in a semicircular-annulus for different nanofluids using our newly proposed nonhomogeneous dynamic mathematical model (Uddin et al. (2016)). The reason of choosing semicircular-annulus shape geometry is that it has huge potentiality in the field of nuclear, civil, mechanical and architectural engineering. Specially, the semicircular-annulus has a comprehensive prospect for use in the cooling channel of nuclear reactor, receiver of solar thermal collector, a duct by which heat can be passed through for the security and safety of the industry, the design of the electronic and electrical equipment for reducing heat. In engineering applications, the geometry that occurs is more complicated than an enclosure containing convective fluids. However, the important insights of heat transfer of the very complex design can be provided by the simple geometry. The geometrical configuration of concern is with the existence of physical structure embedded within the enclosure. Nevertheless, comparatively, little work has been done on natural convection heat transfer in semicircular annuli. As per authors’ knowledge, the literature review revealed that three different boundary conditions taking into nonhomogeneous dynamic mathematical model in a semicircular-annulus shape enclosure filled with nanofluid has not been studied yet. The study has been carried out numerically with an accurate numerical procedure, and the related results have been shown graphically for cobalt-kerosene nanofluid in the form of streamlines, isotherms, and isoconcentrations. Moreover, the heat transfer rates of 12 types of
nanofluids have been displayed in a bar diagram for three different heat sources which is very rare in the heat transfer analysis. Since the enclosure introduced in this paper can have an inordinate use in nuclear reactor and making solar collectors, the numerical analysis presented can help to promote the use of renewable energy.

![Schematic view of the model with boundary conditions.](image)

**Fig. 1.** Schematic view of the model with boundary conditions.

## 2. Problem Formulations

### 2.1 Physical modeling

A schematic geometry of the two-dimensional problem under investigation is shown in Fig. 1, where $x$ and $y$ are the Cartesian coordinates. We have considered a semicircular-annulus filled with nanofluids having inner radius $r_i$ and outer radius $r_o$. The three different thermal boundary conditions of the inner semicircular curve of the annulus is modeled as $T = T_h$, $T = T_c + (T_h - T_c)(x/L - 1/4)(3/4 - x/L)$ and $T = T_c + (T_h - T_c)\sin(4\pi x/L)$, where $L/4 = r_o - r_i$ is the diameter of the semi-annulus. Two other horizontal lines are modeled as adiabatic assuming no heat escapes through these boundaries. The temperature of the outer semicircular boundary of the enclosure is low as heat goes through it and modeled as $T = T_c$. Initially, it is assumed that nanofluid concentration is kept at low concentration $C_c$ but for $t > 0$, it is assumed as $C_i$ in the entire domain so that $C_h > C_c$. Thermophoresis, Brownian diffusion and gravity effects are included in our study in the absence of thermal radiation and chemical reaction. The base fluid and the solid nanoparticles
are thermally equilibrium. Cobalt-kerosene nanofluid as default has been analyzed in terms of flow, thermal and concentrations fields as well as heat transfer enhancement. In addition, 12- types of nanofluids have been considered for the best performers of heat transfer enhancement compared to the base fluids.

2.2 Mathematical modeling

2.2.1 Conservation equations for nanofluids

To construct conservation equations of nanofluids for accurate results we have considered the slip mechanisms. For the present study, we have also considered the Brownian motion of the nanoparticles, thermophoretic diffusion and gravitational effect as slip mechanisms. We assumed that the base fluids and the nanoparticles are in thermally equilibrium, there are no chemical reaction between solids and base fluids, and no radiative heat transfer. Since, the size of nanoparticles is very small and they are easily fluidized, we treated nanofluid as a fluid containing nanoparticles as species of base fluids. As the concentration is very important in nanofluids to understand the denseness as well as stable situation of nanoparticles in the base fluids, we have considered the nanofluids concentration equation that has been neglected in well-known one-component as well as two-component mathematical models. Therefore, under the aforementioned assumptions, the dynamic unsteady conservation equations namely, the continuity, momentum, energy and concentration equations for nanofluids have been constructed as follows (Uddin et al. 2016):

The nanofluid continuity equation:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

The nanofluid momentum equation in \(x\)-direction:

\[
\rho_{nf} \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial p}{\partial x} + \mu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{2}
\]

The nanofluid momentum equation in \(y\)-direction:

\[
\rho_{nf} \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial p}{\partial y} + \mu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + (\rho\beta)_{nf} g(T - T_c) + (\rho\beta^*)_{nf} g(C - C_c) \tag{3}
\]

The nanofluid energy equation:

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + D_b \left( \frac{\partial C}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + D_r \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] \tag{4}
\]

The nanofluid concentration equation:

\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_b \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{CD_l}{T} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + D_r \left( \frac{\partial C}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) \tag{5}
\]

where \(u, v\) are the velocity along \(x, y\) coordinates, respectively; \(p\) is the pressure, \(g\) is the gravity, \(T\) is the temperature, \(T_c\) is the reference temperature, \(C\) is the concentration and \(C_c\) is the reference concentration of nanofluids. Here, \(\mu_{nf}\) is the dynamic viscosity of nanofluid, \(\rho_{nf}\) is the density of nanofluid, \(\alpha_{nf} = \frac{\kappa_{nf}}{(\rho C_p)_{nf}}\) is the thermal diffusivity of nanofluid, \(\kappa_{nf}\) is the thermal conductivity of nanofluid, \((\rho C_p)_{nf}\) is the heat capacity of nanofluid, \((\rho\beta)_{nf}\) is the volumetric thermal
expansion of nanofluid and \((\rho\beta')_n\) is the volumetric mass expansion of nanofluid. The popular relations of effective viscosity, density, heat capacitance, volumetric thermal expansion, and volumetric mass expansion of nanofluid respectively are given by

\[
\mu_{nf} = \mu_{bf}/(1 - \phi)^{2.5} \tag{6}
\]

\[
\rho_{nf} = (1 - \phi)\rho_{bf} + \phi \rho_p \tag{7}
\]

\[
\left(\rho c_p\right)_{nf} = (1 - \phi)(\rho c_p)_{bf} + \phi (\rho c_p)_p \tag{8}
\]

\[
(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_{bf} + \phi (\rho\beta)_p \tag{9}
\]

\[
(\rho\beta')_{nf} = (1 - \phi)(\rho\beta')_{bf} + \phi (\rho\beta')_p \tag{10}
\]

The most popular thermal conductivity models for this type of solid-liquid mixture are the Maxwell model (1873) and Hamilton and Crosser (1962). In these models, the thermal conductivity is defined as follows:

\[
\kappa_{nf} = \frac{2\kappa_{bf} + \kappa_p - 2\phi(\kappa_{bf} - \kappa_p)}{2\kappa_{bf} + \kappa_p + \phi(\kappa_{bf} - \kappa_p)} \tag{11}
\]

\[
\kappa_{nf} = \kappa_{bf} \left[ \frac{\kappa_p + (n-1)\kappa_{bf} - (n-1)\phi(\kappa_{bf} - \kappa_p)}{\kappa_p + (n-1)\kappa_{bf} + \phi(\kappa_{bf} - \kappa_p)} \right] \tag{12}
\]

Equation (12) represents the thermal conductivity with particles shape factor \(n = 3 / \Psi\), where \(\Psi\) is the sphericity defined as the ratio between the surface area of the sphere and the surface area of the real particle with equal volumes. The values of \(\Psi\) are evaluated experimentally 0.81, 0.62, 0.52 and 0.36 for brick, cylinder, platelet and blade shape nanoparticle respectively (see Timofeeva et al., 2009). Hence, Maxwell’s (1873) thermal conductivity equation is defined for the shape factor \(n = 3\), which represents the spherical shape particle in the base fluid and Hamilton and Crosser’s (1962) thermal conductivity of heterogeneous two-component system includes empirical shape factor where \(n = 3.7, n = 4.9, n = 5.7\) and \(n = 8.6\) represents brick, cylinder, platelet and blade shape nanoparticle respectively.

Furthermore, Xuan et al. (2003) considered the Brownian motion of nanoparticles with the Maxwell model and proposed a modified formula for the dynamic effective thermal conductivity model which is as follows:

\[
\kappa_{nf} = \frac{2\kappa_{bf} + \kappa_p - 2\phi(\kappa_{bf} - \kappa_p)}{2\kappa_{bf} + \kappa_p + \phi(\kappa_{bf} - \kappa_p)} + \frac{\left(\rho c_p\right)_p \phi}{2\sqrt{3\pi\mu_{nf} d_p}} \frac{2k_B T_C}{2\sqrt{3\pi\mu_{nf} d_p}} \tag{13}
\]

where, \(k_B\) is the Boltzmann constant, \(d_p\) is the diameter of a nanoparticle. In equation (13), the second term on the right hand side is due to the Brownian motion of nanoparticles.

Therefore, considering shape of the nanoparticles the equation (13) can be written with Hamilton and Crosser’s (1962) thermal conductivity equation as

\[
\kappa_{nf} = \frac{(n-1)\kappa_{bf} + \kappa_p - (n-1)\phi(\kappa_{bf} - \kappa_p)}{(n-1)\kappa_{bf} + \kappa_p + \phi(\kappa_{bf} - \kappa_p)} + \frac{\left(\rho c_p\right)_p \phi}{2\sqrt{3\pi\mu_{nf} d_p}} \frac{2k_B T_C}{2\sqrt{3\pi\mu_{nf} d_p}} \tag{14}
\]

This model (equation (14)) has been used in the present study in order to define the effective thermal conductivity of nanofluids.

The Brownian diffusion coefficient for nanoparticles is defined by,
D_h = \frac{k_p T_C}{3\pi \eta d_p} \quad \text{(15)}

The thermophoretic velocity equation for nanofluids is defined as (see Uddin et al., 2015)

\[ V_T = 0.126 \frac{\kappa_{bf} \chi \beta_{bf} \eta_{bf}}{\rho_{bf}} \nabla (\ln T) \quad \text{(16)} \]

where, \( \beta_{bf} \) is the thermal expansion of the bulk fluid and the symbol \( \chi \) is a correction factor depends on the size and shape of the particles as \( \chi = -0.0002d_p + 0.1537 \). For a particle size of 100 nm, the values of \( \chi \) is found 0.1337 and for the particle size of 1 nm the value of \( \chi \) is 0.1535. The thermal diffusion coefficient, \( D_T \) can be incurred from the thermophoretic velocity equation as (Uddin et al., 2015)

\[ D_T = 0.126 \frac{\kappa_{bf} \chi \beta_{bf} \eta_{bf}}{\rho_{bf}} \quad \text{(17)} \]

Thus, for water and hexane based nanofluids with polystyrene latex nanoparticles at room temperature, thermal diffusion coefficient, \( D_T \) can be obtained from equation (17) as \( D_T = 3.639 \times 10^{-12} \text{m}^2\text{s}^{-1}\text{K}^{-1} \). This result is very consistent with the experimental result of Shiundu et al. (2003) and Iacopini et al. (2006).

### 2.2.2 Boundary conditions

The appropriate initial and boundary conditions for the present problem along with the above stated model are as follows:

For \( t = 0 \), entire domain: \( u = v = 0, \ T = T_C, \ C = C_C, \ p = 0 \) \quad \text{(18a)}

For \( t > 0 \),

On the inner semicircular wall:

Case-1: \( u = v = 0, \ T = T_h, \ C = C_h \) \quad \text{(18b)}

Case-2: \( u = v = 0, \ T = T_C + (T_h - T_C)(x/L - 1/4)(3/4 - x/L), \ C = C_h \) \quad \text{(18c)}

Case-3: \( u = v = 0, \ T = T_C + (T_h - T_C) \sin(4\pi x/L), \ C = C_h \) \quad \text{(18d)}

On the outer semicircular wall: \( u = v = 0, \ T = T_c, \ C = C_h \) \quad \text{(18e)}

On the two horizontal insulated walls: \( u = v = 0, \ \frac{\partial T}{\partial n} = 0, \ C = C_h \) \quad \text{(18f)}

### 2.2.3 Non-dimensional governing equations

To describe several transport mechanisms in nanofluids, it is meaningful to make the conservation equations non-dimensional. The main benefits of non-dimensionalization are (i) one can easily understand the controlling flow parameters of the system, (ii) get rid of dimensional constraint and (iii) make a generalization of the size and shape of the geometry. For this purpose, the above equations can be converted to non-dimensional forms, using the following dimensionless quantities:

\[ U = \frac{u L}{\alpha_{bf}}, V = \frac{v L}{\alpha_{bf}}, X = \frac{x}{L}, Y = \frac{y}{L}, \theta = \frac{T - T_C}{\Delta T}, P = \frac{p L^2}{\rho_{bf} \alpha_{bf}}, \xi = \frac{\alpha_{bf} t}{L}, \Phi = \frac{C - C_c}{\Delta C} \quad \text{(19)} \]
where, $\alpha_{bf}$, $L$, $\Delta T = T_h - T_c$, $\Delta C = C_h - C_c$ and $T_c$ and $C_c$ are the thermal diffusivity of the base fluid, reference length of the geometry, the temperature difference, the nominal concentration difference, reference temperature and reference concentration within the nanofluid, respectively. Introducing the above transformations into the conservation equations (1)-(5), assuming constant properties of nanofluids and recognizing that $(\Delta T / T_c) \ll 1$ and $(\Delta C / C_c) \ll 1$, the governing equations (1)-(5) can be written in dimensionless forms as:

**Continuity equation:**
\[
\frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial \eta} = 0
\]  

**Momentum equation in $X$ - direction:**
\[
\frac{\partial U}{\partial \xi} + U \frac{\partial U}{\partial \xi} + V \frac{\partial U}{\partial \eta} = - \frac{\rho_{bf} \partial P}{\rho_{bf} \partial \xi} + \frac{\mu_{bf}}{v_{bf} \rho_{bf}} \text{Pr} \left( \frac{\partial^2 U}{\partial \xi^2} + \frac{\partial^2 U}{\partial \eta^2} \right)
\]

**Momentum equation in $Y$ - direction:**
\[
\frac{\partial V}{\partial \xi} + U \frac{\partial V}{\partial \xi} + V \frac{\partial V}{\partial \eta} = - \frac{\rho_{bf} \partial P}{\rho_{bf} \partial \eta} + \frac{\mu_{bf}}{v_{bf} \rho_{bf}} \text{Pr} \left( \frac{\partial^2 V}{\partial \xi^2} + \frac{\partial^2 V}{\partial \eta^2} \right) + \frac{(\rho \beta)}{\beta_{bf} \rho_{bf}} \text{Ra}_T \text{Pr} \theta + \text{Ra}_C \text{Pr} \Phi
\]

**Energy equation:**
\[
\frac{\partial \theta}{\partial \xi} + U \frac{\partial \theta}{\partial \xi} + V \frac{\partial \theta}{\partial \eta} = \frac{\alpha_{bf}}{\text{Pr}} \left( \frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2} \right) + \frac{\rho \beta}{\beta_{bf} \rho_{bf}} \text{Pr} \left( \frac{\partial \theta}{\partial \xi} \right)^2 + \left( \frac{\partial \theta}{\partial \eta} \right)^2
\]

**Concentration equation:**
\[
\frac{\partial \Phi}{\partial \xi} + U \frac{\partial \Phi}{\partial \xi} + V \frac{\partial \Phi}{\partial \eta} = \frac{\text{Pr}}{\text{Sc}} \left( \frac{\partial^2 \Phi}{\partial \xi^2} + \frac{\partial^2 \Phi}{\partial \eta^2} \right) + \frac{1}{\text{Sc}} \left( \text{N}_{TTR} \left( \frac{\partial \Phi}{\partial \xi} \right)^2 + \left( \frac{\partial \Phi}{\partial \eta} \right)^2 \right)
\]

The dimensionless parameters introduced in the above equations (20)-(24) are as follows:

- $\text{Ra}_T = \frac{L^3 \beta_{bf} g \Delta T}{v_{bf} \alpha_{bf}}$ is the local thermal Rayleigh number,
- $\text{Ra}_C = \frac{L^3 \rho \beta (\rho \beta')_{bf}}{v_{bf} \alpha_{bf} \rho_{bf}}$ is the local solutal Rayleigh number,
- $\text{Pr} = \frac{v_{bf}}{\alpha_{bf}}$ is the Prandtl number,
- $\text{Le} = \frac{\kappa_{bf} C_c}{(\rho c_p)_{bf} D_b \Delta C}$ is the modified Lewis number,
- $\text{N}_{TTR} = \frac{D_f \Delta T}{D_b T_c}$ is the dynamic thermo-diffusion parameter,
- $\text{N}_{TRC} = \frac{D_f \Delta T C_c}{D_b T_c}$ is the dynamic diffusion parameter and $\text{Sc} = \frac{\mu_{bf}}{\rho_{bf} D_b}$ is the Schmidt number.

### 2.2.4 Non-dimensional boundary conditions

The initial and boundary conditions in the dimensionless form for the present problem can be written as:

**For $\xi = 0$, entire domain:** $U = V = 0, \quad \theta = 0, \quad \Phi = 0, \quad P = 0$  

**For $\xi > 0$,**

\[ (25a) \]
On the inner circular wall:

Case-1: \( U = V = 0, \quad \theta = 1, \quad \Phi = 1 \)  

Case-2: \( U = V = 0, \quad \theta = \left( X - \frac{1}{4} \right) \left( \frac{3}{4} - X \right), \quad \Phi = 1, \quad 0.25 \leq X \leq 0.75 \)  

Case-3: \( U = V = 0, \quad \theta = \sin(4\pi X), \quad \Phi = 1, \quad 0.25 \leq X \leq 0.75 \)  

On the outer circular wall: \( U = V = 0, \quad \Phi = 1 \)  

On the two horizontal insulated walls: \( U = V = 0, \quad \frac{\partial \theta}{\partial n} = 0, \quad \Phi = 1 \)  

3. Thermophysical Properties

The thermophysical properties of nanofluids directly depend on the thermophysical properties of nanoparticles and base fluids. Thermophysical properties of base fluids and nanoparticles can be simply defined as the characteristics of fluids system as well as the material properties these varies with the state variables temperature and pressure without altering their chemical identity. For our current nanofluid research, we use specific heat capacity, density, thermal conductivity and volumetric thermal expansion coefficients of base fluids and particles, as well as the viscosity of the base fluids at room temperature. These thermophysical properties are shown in Table 1 (Oztop and Abu-Nada, 2008; Rahman and Aziz, 2012; Mutuku, 2014).

4. Nusselt Number and Temperature Gradient Magnitude

The local nusselt number on the heat source surface can be expressed as

\[
Nu_L = -\frac{\kappa_{nf}}{\kappa_{bf}} \frac{\partial \theta}{\partial Y}
\]  

The average Nusselt number is evaluated by integrating \( Nu_L \) along the heat source

\[
Nu_{ave} = -\frac{\kappa_{nf}}{\kappa_{bf}} \frac{4}{\pi} \int_0^{\pi/4} \frac{\partial \theta}{\partial Y} dX
\]  

The temperature gradient magnitude of the flow of nanofluid can be defined as

\[
GM_T = \sqrt{\left( \frac{\partial \theta}{\partial X} \right)^2 + \left( \frac{\partial \theta}{\partial Y} \right)^2}
\]
Table 1 Thermophysical properties of base fluids and materials at room temperature.

<table>
<thead>
<tr>
<th>Items</th>
<th>$c_p$ (Jkg$^{-1}$K$^{-1}$)</th>
<th>$\rho$ (kgm$^{-3}$)</th>
<th>$\kappa$ (Wm$^{-1}$K$^{-1}$)</th>
<th>$\mu$ (kgm$^{-1}$s$^{-1}$)</th>
<th>$\beta$ (K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water (H$_2$O)</td>
<td>4179</td>
<td>997.1</td>
<td>0.613</td>
<td>0.001003</td>
<td>21×10$^{-5}$</td>
</tr>
<tr>
<td>Kerosene</td>
<td>2090</td>
<td>780</td>
<td>0.149</td>
<td>0.00164</td>
<td>99×10$^{-5}$</td>
</tr>
<tr>
<td>Al$_2$O$_3$</td>
<td>765</td>
<td>3970</td>
<td>40</td>
<td>–</td>
<td>0.85×10$^{-5}$</td>
</tr>
<tr>
<td>TiO$_2$</td>
<td>686.2</td>
<td>4250</td>
<td>8.9538</td>
<td>–</td>
<td>0.9×10$^{-5}$</td>
</tr>
<tr>
<td>CuO</td>
<td>531.8</td>
<td>6320</td>
<td>76.5</td>
<td>–</td>
<td>1.8×10$^{-5}$</td>
</tr>
<tr>
<td>ZnO</td>
<td>495.04</td>
<td>5610</td>
<td>29</td>
<td>–</td>
<td>4.7×10$^{-5}$</td>
</tr>
<tr>
<td>Fe$_3$O$_4$</td>
<td>670</td>
<td>5180</td>
<td>80.4</td>
<td>–</td>
<td>20.6×10$^{-5}$</td>
</tr>
<tr>
<td>Cobalt</td>
<td>420</td>
<td>8900</td>
<td>100</td>
<td>–</td>
<td>1.3×10$^{-5}$</td>
</tr>
</tbody>
</table>

5. Computational Procedures

The Galerkin weighted residual scheme of finite element method (FEM) has been applied to solve the governing dimensionless equations (20)-(24) together with the boundary conditions (25). The details with the benefits of this numerical method are well analyzed by Zienkiewicz et al. (2005), Codina (1998) and Al Kalbani et al. (2016). In this method, the solution domain is discretized into finite element meshes, which are composed of non-uniform triangular elements. Six node triangular elements are used in this work for the development of the finite element equations. All six nodes are associated with velocities, temperature as well as isoconcentration; only the corner nodes are associated with pressure. This means that a lower order polynomial is chosen for pressure and which is satisfied through the continuity equation. Then the nonlinear governing partial differential equations (i.e. conservation of mass, momentum, energy and concentration equations) are transferred into a system of integral equations by applying Galerkin weighted residual method. The integration involved in each term of these equations is performed by using Gauss’s quadrature method. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. To solve the set of the global nonlinear algebraic equations in the form of matrix, the Newton-Raphson iteration technique has been adapted through partial differential equation solver with MATLAB interface. The convergence criterion of the numerical solutions along with error estimation has been set to $\|I^{m+1} - I^m\| \leq 10^{-5}$, where $I$ is the general dependent variable ($U$, $V$, $\theta$, $\Phi$) and $m$ is the number of iteration.
Fig. 2. Grid sensitivity test for cobalt-kerosene nanofluid with $Ra_r = 10^5$, $d_p = 10\text{ nm}$, $\phi = 0.05$ at $\xi = 1$.

5.1 Grid independency test

To test and assess grid independence of the solution for cobalt-kerosene nanofluid, an extensive mesh testing technique has been conducted for case-3 with the selected values of $Ra_r = 10^5$, $Ra_c = 10^3$, $\phi = 0.05$, $d_p = 10\text{ nm}$, $n = 3$ (spherical shape nanoparticle) at $\xi = 1$. In the present work, we inspect six different non-uniform grid systems with the following number of elements within the resolution field: 1276, 2012, 3064, 9080, 24020 and 33372. The numerical design is carried out for highly precise key in the average Nusselt number ($Nu_{ave}$) at the heated inner semicircular wall for the aforesaid elements to develop an understanding of the grid fineness as shown in Fig. 2. As can be seen that the average Nusselt number enhances, as the number of triangular elements of the geometry increases up to a certain stage and after that it becomes plateau. These indicate that after generating certain number of elements in the mesh, all the results are independent of mesh sizes. In the present study, the scale of $Nu_{ave}$ for 24020 elements shows a very little difference with the results obtained for the elements of 33372. Hence, from the numerical experiment, 24020 elements are found to meet the requirement of the grid independency study.

5.2 The flow parameters

The parameters of the flow are calculated by their definition with the thermophysical properties of the nanoparticles and base fluids. The values of the Prandtl number are calculated for water and kerosene as 6.8377 and 23.004 respectively. Let us consider $\Delta T = 10\text{K}$, $\Delta C = 0.01$, $T_c = 300\text{K}$, $C_c = 1$, $d_p = 10\text{ nm}$, $n = 3$ and $\phi = 0.05$. With these values, the physical parameters entered into the
equations (20)-(24) for cobalt-kerosene nanofluid can be calculated as follows: \( Le = 3.8793 \times 10^5 \), \( Sc = 89239 \), \( D_p = 2.3561 \times 10^{-11} \), \( D_T = 4.0352 \times 10^{-11} \), \( N_{TBTCN} = 5.7089 \) and \( N_{TBTN} = 0.057089 \). The other parameters such as the Rayleigh number \( (Ra_T) \), the solutal Rayleigh number \( (Ra_C) \) are varied to analyze the flow and thermal characteristics of a specific nanofluid. As the thermophysical properties of nanofluids depend on the thermophysical properties of nanoparticles and base fluids, the values of the above stated parameters will be different for each nanofluid. It is important to note that as the Brownian and thermophoretic diffusions strongly depend on the diameters of nanoparticles, the Brownian diffusion coefficient \( (D_B) \) and the thermal diffusion coefficient \( (D_T) \) changes with the size of the nanoparticles. Also, the values of the parameters especially \( Le \), \( Sc \), \( N_{TBTCN} \) and \( N_{TBTN} \) will change according to the size of the particles, nanoparticles volume fraction and thermophysical properties of nanofluids.

7. Results and Discussion

In this section, the obtained numerical results for natural convective heat transfer of nanofluids in a semicircular-annulus shaped enclosure with three different thermal boundary conditions (case-1: uniform, case-2: parabolic and case-3: sinusoidal distribution of temperature) at the inner circular wall are discussed. The physical phenomenon is investigated for a wide range of controlling parameters. In the numerical simulation, we have considered cobalt- kerosene nanofluid as default for the description of the flow, thermal and concentration fields as well as average Nusselt number for different model parameters of the problem. The most important features of nanofluids are the effect of nanoparticle in terms of amount, size and shape, are also investigated and displayed graphically. The ratio of the buoyancy driven parameters \( Ra_T \) and \( Ra_C \) are kept fixed to \( 10^5 \). The results are analyzed varying nanoparticle volume fraction \( (\phi = 0.0, 0.05, 0.10) \), nanoparticle size \( (d_p = 1, 20, 30 \text{ nm}) \) and Rayleigh numbers \( (Ra_T = 10^5, \text{to} \ Ra_T = 10^7) \) on streamlines, isotherms and isoconcentrations. The average Nusselt number and temperature gradient magnitude versus \( X \), on the inner circular hot wall and outer colder wall for three different cases are displayed as line graphs. Finally, the average Nusselt number of 12 types of nanofluids for different solid volume fraction for three different thermal boundary cases has been displayed in a bar diagram. A brief albeit sufficient explanation follows each result.

Isotherms are helpful to detect the effectiveness of heat transfer in a fluid and also tell us about the dictating mode of heat transfer whether it is strong convection or weak convection. Figure 3 represents the effect of solid volume fraction on isotherms for three different boundary conditions on the inner semicircular wall with \( n = 3 \), \( d_p = 10 \text{ nm} \), \( Ra_T = 10^5 \) at \( \xi = 1 \). As can be seen that there are mushroom like isotherm distributions from the inner wall to the upper curved wall and as nanoparticle volume fraction increases, the dissemination of isotherms little bit compresses to the inner hot wall throughout the cases. It is also interesting to note that isotherms are symmetrically distributed relative to the annulus for both case-1 and case-2 whereas, two mushroom like asymmetric isotherms spread in the annulus for case-3. However, the pattern of isotherms by case-1 and case-3 are more dispersive, full-bodied and distorted in the middle of the annulus than that of case-2. This indicates, convection mode of heat transfer is higher and stronger for case-1 and case-3 than that of case-2. For a relative comparison of isotherms between using case-1 and case-3, case-3 gives more distorted isotherms than case-1 which is a clear indication that higher heat transfer rate
occurs for case-3. As case-3 has significant potentiality to rise augmentation of heat transfer rate, this condition is worthwhile to understand the convection, flow field and concentration levels of nanofluid in the annulus.

![Figure 3](image)

**Fig. 3.** Effect of $\phi$ on isotherms for selected values of $d_p = 10 \text{ nm}$, $Ra_f = 10^7$, $n = 3$ at $\xi = 1$ for three different thermal boundary conditions.

The effect of nanoparticle diameter on isoconcentrations for selected values of $Ra_f$ with $\phi = 0.05$, $n = 3$ at $\xi = 1$ has been displayed in Fig. 4. As can be seen that isoconcentrations of Co-kerosene nanofluid represented by three loops in the annulus due to the sinusoidal boundary conditions. Nanoparticle diameter determines the concentration levels of nanofluid in the annulus. As nanoparticle diameter increases, the isoconcentrations in the loops become weaker which is an indication of lower heat transfer rate for higher values of particle diameter. Furthermore, density of isoconcentrations decreases when particle diameter increases. It can also be seen that as particle diameter exceeds 20 nm, isoconcentrations clusters on the walls and not uniform in the annulus. The strength and density of isoconcentrations in the middle of the annulus slightly increases for $d_p > 20 \text{ nm}$ whereas, remarkable enhancement can be found for particle diameter 1-20 nm, as buoyancy driven parameter increases. These may be due to the higher Brownian diffusion for lower diameter of particle. It is a good indication from this figure that nanoparticles in the base fluid become uniform and stable in the semicircular annulus for particle diameter within the range of 1-20 nm.
Fig. 4. Effect of $d_p$ on isoconcentrations for selected values of $Ra_T$ with $\phi = 0.05$, $n = 3$ at $\xi = 1$.

Fig. 5. Effect of $d_p$ on isoconcentrations for selected values of $Ra_T$ with $\phi = 0.05$, $n = 3$ at $\xi = 10$. 

Figure 5 illustrates the continuation effect of the nanoparticles diameter on isoconcentration for the time domain, $\xi = 10$ (steady state). It is clearly seen that strength of isoconcentration increases significantly than it was in the time, $\xi = 1$ (unsteady state). The density of isoconcentration decreases as the nanoparticles diameter and buoyancy driven parameter, Rayleigh number increases. The similar results are found for the nanoparticles diameter, $d_p > 20$ nm, as it was in the time domain, $\xi = 1$. It can be found from the figure that for the particle diameter, $d_p = 30$ nm, the isoconcentrations become significantly weaker compared to other isoconcentrations for nanoparticles diameter (1-20 nm). So we can summarize that nanofluid holding 1-20 nm size particles are uniform and stable solution in the semicircular annulus.

Figure 6 depicts the influence of adding nanoparticles to the fluid inside the annulus by showing streamline maps against $Ra_T$ with $d_p = 10$ nm, $n = 3$ at $\xi = 1$. It is seen that there are three rotating vortices inside the annulus where two vortices near the corner of the annulus rotating clockwise and other one rotating counterclockwise in the middle. This pattern is the result of thermal boundary condition at the inner wall. As buoyancy driven parameter, $Ra_T$ increases, distributions of streamlines are found in the entire annulus, that is, a full bodied flow occurs. The strength of the fluid currents increases with $Ra_T$; as higher $Ra_T$ results in higher convective force. Another interesting observation is, as nanoparticle volume fraction increases, vortices inside the annulus slightly compresses and density of streamlines increases. These phenomena can be described by the fact that nanoparticles increase the total mass of the fluid which is reflected as higher inertia of the fluid. This higher inertia is causing the fluid to slow down a bit.

**Fig. 6.** Effect of $\phi$ on streamlines for selected values of $Ra_T$ for case-1 with $d_p = 10$ nm, $n = 3$ at $\xi = 1$. 

\begin{center}
\begin{tabular}{ccc}
0.0 & \multicolumn{2}{c}{0.05} \\
0.1 & \multicolumn{2}{c}{0.1} \\
$\phi$ & $Ra_T = 10^5$ & $Ra_T = 10^6$ & $Ra_T = 10^7$
\end{tabular}
\end{center}
Figure 7 shows the isotherm pattern for the present problem under different values of $Ra_T$ and nanoparticle volume fraction, $\phi$ with $d_p = 10\,\text{nm}$, $n = 3$ and case-1 at $\xi = 1$. For the base fluid; the isotherms are more dispersive and almost parallel to each other in the entire annulus. As nanoparticle volume fraction increases, isotherms are stronger, rigid and not parallel in most of the region of the annulus. This is due to heat diffusion by nanoparticles. Furthermore, the isotherm pattern seems to indicate that at low value of $Ra_T$, convection is weaker inside the cavity as two loops of isotherms are seen almost parallel to each other near the heated inner wall. As the value of $Ra_T$ increases, the isotherms become more and more distorted and forming two distorted mushroom like isotherm patterns in the two sides of the annulus. This particular pattern suggests that heat energy is flowing strongly into the fluid from the inner heated curved wall and convection mode of heat transfer dominates.

![Isotherms](image)

**Fig. 7.** Effect of $\phi$ on isotherms for selected values of $Ra_T$ for case-1 with $d_p = 10\,\text{nm}$, $n = 3$ at $\xi = 1$.

The effect of solid volume fraction on isoconcentrations for selected values of $Ra_T$ for case-1 with $d_p = 10\,\text{nm}$, $n = 3$ at $\xi = 1$ has been portrayed in Fig. 8. It is observed that isoconcentrations are more or less uniform for all cases in the entire semicircular annulus; however, isoconcentrations of nanofluid are slightly more noticeable than base fluid. As the nanoparticle volume fraction increases the level of isoconcentrations also increases. This is due to the more interactions among particles in fluid. The strength of the isoconcentrations increases when $Ra_T$ increases. It is interesting to see that the effect of solid volume fractions on the pattern of isoconcentrations in annulus is almost similar to that of streamlines. This kind of outcome is very rare in other types of enclosure. This clearly indicates that annulus shape enclosure plays very important role to obtain full-bodied flow and uniform isoconcentrations of nanofluid.
Fig. 8. Effect of $\phi$ on isoconcentrations for selected values of $Ra_T$ with $d_\rho = 10$ nm, $n = 3$, at $\xi = 10$.

Figures 9(a)-9(c) and Figures 10(a)-10(c) respectively, show variation of local Nusselt number along the inner heated curve wall and upper relatively colder wall for different solid volume fraction using (a) case-1, (b) case-2, and (c) case-3 thermal conditions with $d_\rho = 10$ nm, $Ra_T = 10^7$, $n = 3$ at $\xi = 1$. It is observed from the figures that local Nusselt number on both heated and relatively colder wall is remarkably higher for nanofluid than that of base fluid. It is also found that if there is more nanoparticles, there will be more heat transfer rate. From Fig.9 (a), it is found that distribution of local Nusselt number at the heated wall for nanofluid is almost a parabola opening upwards whereas, it is U-shaped for base fluid with axis of symmetry at $X = 0.5$. In this case, the highest value of local Nusselt number is found at the corners of the annulus. However, the distribution of local Nusselt number at upper colder wall for nanofluid is a complete parabola opening downward and for base fluid, it is almost a straight line with a little bump in the middle (Fig. 10 (a)). In this case, the maximum value of local Nusselt number is found at the middle of the upper curved wall. These distinguishable phenomena are due to nanoparticles in the solution where nanoparticles carry significant amount of heat from bottom wall to upper colder wall. Fig. 9(b) represents the M shape distribution of local Nusselt number with two peaks at the two sides of the heated wall, though there is a quadratic thermal boundary condition. Nevertheless, at the outer cold wall, 10(b) also shows parabolic distribution of local Nusselt number. Interesting results are found for case-3. From Fig. 9(c) and Fig. 10(c), it is demonstrated that local Nusselt number is noticeably higher for nanofluid than that of base fluid compare to other two cases. Local Nusselt number is maximum for nanofluid on the hotter wall at $X = 0.55$ and on the outer wall at $X = 0.8$; which clearly indicates that heat is diffused by nanoparticle in the annulus. The distribution of local Nusselt is strongly wavy for nanofluid than that of base fluid. These clearly indicate that heat transfer rate is higher for sinusoidal thermal boundary condition than other two cases.
Fig. 9. Effect of local Nusselt number on the inner hot round wall for (a) case-1, (b) case-2 and (c) case-3 for $d_p = 10$ nm, $Ra_T = 10^7$, $n = 3$ at $\xi = 1$.

Fig. 10. Effect of local Nusselt number on the outer cold round wall for (a) case-1, (b) case-2 and (c) case-3 for $d_p = 10$ nm, $Ra_T = 10^7$, $n = 3$ at $\xi = 1$. 
Fig. 11. Effect of temperature gradient magnitude on the inner hot round wall for (a) case-1, (b) case-2 and (c) case-3 for $d_p = 10$ nm, $Ra_f = 10^7$, $n = 3$ at $\xi = 1$.

Fig. 12. Effect of temperature gradient magnitude on the outer cold round wall for (a) case-1, (b) case-2 and (c) case-3 for $d_p = 10$ nm, $Ra_f = 10^7$, $n = 3$ at $\xi = 1$. 
The average Nusselt number versus (a) nanoparticle diameter for $Ra_T = 10^7$, $n = 3$ (b) Rayleigh number with $d_p = 10 \text{ nm}$, $n = 3$ and (c) shapes of nanoparticle for $d_p = 10 \text{ nm}$, $Ra_T = 10^7$ using case-3 at $\xi = 1$.

The variation of temperature gradient magnitude ($GM_T$) at the inner hot and outer cold wall for solid volume fraction using (a) case-1, (b) case-2 and (c) case-3 thermal boundary conditions with $d_p = 10 \text{ nm}$, $Ra_T = 10^7$, $n = 3$ at $\xi = 1$ has been displayed in Fig. 11(a)-11(c) and Fig. 12(a)-12(c) respectively. It is observed that the temperature gradient magnitude at the hotter or colder wall is higher for base fluid than that of nanofluid throughout the cases. As nanoparticle volume fraction increases, temperature gradient magnitude on the hotter and relatively colder wall decreases which is completely a reverse effect of local Nusselt number. This is due to the thermophysical properties of nanofluid and base fluid. So, it can be concluded that the variation of local Nusselt number with respect to the solid volume fraction is inversely proportional to the variation of temperature gradient magnitude alone at the hotter or colder wall. For case-1, as can be seen that the distribution of temperature gradient magnitude is a necklace shape for base fluid and a parabola shape for nanofluid.
opening upward at hotter surface, whereas, at the colder surface, it is also a downward parabola. It is seen from Fig. 11(b) that at the hotter wall, temperature gradient magnitude is an M shape for base fluid and as solid volume fraction increases, it turns to be a U shape. Fig. 12(b) represents a \( \Lambda \) shape temperature gradient distribution at the outer colder wall. Using case-3 thermal boundary condition, it is found from Fig. 11(c) and Fig. 12(c) that at certain points, temperature gradient magnitude of base fluid is equal to that of nanofluid which is completely different from other cases. The variation of temperature gradient magnitude of base fluid and nanofluid is comparatively close to each other in this case.

![Graph](image)

**Fig. 14.** Average Nusselt number enhancement in different water and kerosene based nanofluids for \( \phi = 5\% \) with the selected values of \( Ra_T = 10^7, d_p = 10 \text{nm}, n = 3 \) at \( \xi = 1 \).

Figure 13 displays the average Nusselt number (\( Nt_{\text{ave}} \)) for different flow parameters entered into the problems using case-3 thermal boundary condition. It clearly indicates from Fig.13 that average Nusselt number significantly increases for the increase of the solid volume fraction. Fig.13 (a) represents the average Nusselt number versus the diameter of the nanoparticle for different solid volume fraction. This figure exhibits that average Nusselt number decreases significantly, as diameter of the particle increases. For the particle diameter 1 to 20 nm, a swift decrease in average Nusselt number has been observed and for \( d_p > 20 \text{nm} \), it is decreased steadily for different solid volume fraction. Moreover, the decline rate of average Nusselt number increases significantly with the increase of nanoparticle diameter as solid volume fraction increases. Fig.13 (b) depicts average Nusselt number versus different values of the Rayleigh number for solid volume fraction. Average
Nusselt number increases significantly for the increasing values of the solid volume fraction for different Rayleigh number compared to the base fluid. Also, as buoyancy driven parameter rises, the average Nusselt number rises. Average Nusselt number is almost remain steady from $Ra_T = 10^4$ to $Ra_T = 10^5$ and after that it is gradually increased up to $Ra_T = 10^6$. A dramatic enhancement of average Nusselt number has been found after $Ra_T = 10^6$. From these change it may be concluded that convection and conduction mode of heat transfer can be found from $Ra_T = 10^5$ to $Ra_T = 10^6$ and after that convection mode of heat transfer dominates in the annulus.

Only spherical shape of nanoparticles has been considered for displaying the afore-mentioned results. To justify the shape matters on the heat transfer enhancement of nanofluid, we have considered various shapes of nanoparticles in the present study. The average Nusselt number with respect to nanoparticle volume fraction for different shapes such as sphere, brick, cylinder, platelet and blade of the cobalt (Co) nanoparticles are displayed in Fig. 13(c). From figure, it is clear that the average Nusselt numbers are significantly higher for the blade shape nanoparticles than the spherical ones. This is due to the less sphericity of the blade shape particles; that means higher total surface area of the blade shape solid-liquid interface compared to the total surface area of the spherical shape nanoparticle-liquid interface for the same amount of nanoparticles volume fraction. From this figure, a sequence of higher performer to lower performer of heat transfer with respect to shapes of the nanoparticle is blade, platelet, cylinder, brick and sphere respectively.

For more wide-ranging analysis, the average Nusselt number enhancement compared to base fluid on the heated wall for three different thermal boundary cases of several water-based and kerosene-based nanofluids have been calculated and displayed in Fig. 14 for the selected values of, $Ra_T = 10^7$, $\phi = 0.05$, $d_p = 10 \text{nm}$, $n = 3$ at $\xi = 1$. At a first glimpse, it can be seen that kerosene-based nanofluids have higher heat transfer rate than water based nanofluids. It is also depicted that the enhancement rate of average Nusselt number for cobalt-kerosene nanofluid is higher than other 11 types of nanofluids which are studied in the present analysis. Also, ZnO-water nanofluid has lower heat transfer rate than others. Cobalt-water nanofluid has highest heat transfer rate among water based nanofluids. For water based nanofluids, case-2 and case-3 thermal boundary conditions have equal amount of potentiality to enhance heat transfer rate, whereas case-1 and case-3 thermal boundary conditions play a significant role for kerosene based nanofluids. Specifically, for kerosene based nanofluids, higher heat transfer rates are found using case-3 thermal boundary condition.

### 7. Conclusions

In heat transfer studies, semicircular-annulus shape enclosure is relatively new. Besides, kerosene based nanofluids containing cobalt and Fe$_3$O$_4$ are gaining notice due to their improved physical properties, heat transfer rates and relatively lower cost. In the present study, the analysis of convective unsteady flow of nanofluids in semicircular-annulus shape enclosure has been investigated numerically using nonhomogeneous dynamic model for the understanding of heat transfer mechanisms. All the numerical results are discussed from the physical point of view and the major findings of the study can be listed as follows:
Convection and conduction modes of heat transfer occurs when \( Ra_T \) ranges from \( 10^5 \) to \( 10^6 \) and after that convection mode of heat transfer dominants in the annulus.

1-20 nm size nanoparticles are uniform and stable in the solution.

Semicircular-annulus shape enclosure plays a very important role to obtain a full-bodied flow and uniform isoconcentrations of nanofluid.

The non-uniform sinusoidal heating of the inner curved wall exhibits a higher local Nusselt number at the right regime of the annulus.

Variation of local Nusselt number with respect to the solid volume fraction is inversely proportional to the variation of temperature gradient magnitude.

Average Nusselt number for a base fluid is much lower than a nanofluid. Average Nusselt number decreases significantly, as diameter of the particle increases. For particle diameter 1 to 20 nm, a swift decrease in average Nusselt number has been observed and for \( d_p > 20 \) nm, it is decreased steadily for different solid volume fraction. Moreover, average Nusselt number is higher for blade shape of nanoparticles than other shapes.

Kerosene based nanofluids have higher heat transfer rate than that of water based nanofluids.

Heat transfer rate of cobalt-kerosene nanofluid is higher than other 11 types of nanofluids which are studied in the present analysis. Quadratic and sinusoidal heating of inner wall exhibits equal amount of heat transfer enhancement rate for water based nanofluids which is significantly higher when inner wall heats uniformly. On the other hand, sinusoidal and uniform heating inner curved boundary gives substantially higher heat transfer enhancement rate for kerosene based nanofluids.

**Nomenclature**

\( C \) concentration of nanofluid (mol m\(^{-3}\))

\( C_C \) reference concentration (mol m\(^{-3}\))

\( (c_p)_p \) nanoparticles specific heat (J kg\(^{-1}\) K\(^{-1}\))

\( D_B \) Brownian diffusion coefficient (m\(^2\) s\(^{-1}\))

\( d_p \) diameter of nanoparticle (nm)

\( D_T \) thermal diffusion coefficient (m\(^2\) s\(^{-1}\))

\( g \) acceleration due to gravity (m s\(^{-2}\))

\( h \) heat transfer coefficient of nanofluid (W m\(^{-2}\) K\(^{-1}\))

\( k_B \) Boltzmann constant (JK\(^{-1}\))

\( L \) characteristic length (m)

\( Le \) Lewis number

\( n \) empirical nanoparticle shape factor

\( N_{BTC} \) dynamic diffusion parameter
\( N_{\text{TDF}} \) dynamic thermo-diffusion parameter
\( \text{Nu}_{\text{ave}} \) average Nusselt number
\( \text{Nu}_L \) local Nusselt number
\( p \) dimensional modified pressure (Pa)
\( P \) dimensionless modified pressure
\( \text{Pr} \) Prandtl number
\( \text{Ra}_T \) thermal Rayleigh number
\( \text{Ra}_C \) solutal Rayleigh number
\( \text{Sc} \) Schmidt number
\( t \) dimensional time (s)
\( T \) nanofluid temperature (K)
\( T_c \) reference temperature (K)
\( U, V \) dimensionless nanofluid velocity
\( (u, v) \) dimensional nanofluid velocity (ms\(^{-1}\))
\( V_T \) thermophoretic velocity (ms\(^{-1}\))
\( X, Y \) dimensionless coordinates

**Greek symbols**
\( \alpha_{bf} \) thermal diffusivity of base fluid (m\(^2\)s\(^{-1}\))
\( \alpha_{nf} \) thermal diffusivity of nanofluid (m\(^2\)s\(^{-1}\))
\( \beta_{bf} \) thermal expansion coefficient for base fluid (K\(^{-1}\))
\( \beta_{nf} \) thermal expansion coefficient of nanofluid (K\(^{-1}\))
\( \beta_{nf}^* \) mass expansion coefficient of nanofluid (mol\(^{-1}\))
\( \phi \) nanoparticles volume fraction
\( \Phi \) dimensionless concentration
\( \Psi \) sphericity of the nanoparticle
\( \text{Co} \) cobalt nanoparticle
\( \mu_{bf} \) viscosity of base fluid (kg m\(^{-1}\) s\(^{-1}\))
\( \mu_{nf} \) viscosity of nanofluid (kgm\(^{-1}\)s\(^{-1}\))
\( \nu_{bf} \) kinematic viscosity of base fluid (m\(^2\)s\(^{-1}\))
\( \kappa_{bf} \) base fluid thermal conductivity (Wm\(^{-1}\)K\(^{-1}\))
\( \kappa_{nf} \) nanofluid thermal conductivity (Wm\(^{-1}\)K\(^{-1}\))
\( \kappa_p \) nanoparticle thermal conductivity (Wm\(^{-1}\)K\(^{-1}\))
\( \Delta C \) film concentration drop (mol\(^{-1}\))
\( \Delta T \) film temperature drop (K)
\( \theta \) dimensionless temperature
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Conflict of Interest

None

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