1-D Model for Mass Transfer Calculation in Vortex Tube using Heat and Mass Transfer Analogy

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Abstract

This work introduces a new mathematical model that predicts the mass transfer in a counter current Ranque Hilsch Vortex Tube. The model requires experimental thermal gradient and calculates axial concentration gradient of the heavier species of a binary mixture as mass transfer takes place between two parallel streams moving from inlet towards the hot outlet. The well established Chilton Colburn analogy is used within its applicable range to determine the mass transfer coefficient based on the heat transfer coefficient. The heat transfer coefficient is calculated using Seider-Tate correlation. The model can predict axial concentration gradient of the heavier species in both enriched and depleted stream along the length of the Ranque Hilsch Vortex Tube and mass separation factor. The model is validated with data obtained from experiments conducted with three different vortex generators with variable inlet flow rate. Comparison of the predictions from the model and experimental results shows that the new model can predict the experimental results quite well for a range of flow rate from 0.01 m$^3$/s to 0.03 m$^3$/s and hot end valve opening values ranging from 1.0 to 4.5 turns. The variation of mass separation factor is also examined with inlet feed flow rate and hot end valve opening values. Air as a binary mixture of nitrogen and oxygen is considered as the working fluid.

Keywords: Vortex tube; Species separation; 1-d mathematical model; Heat and mass transfer analogy

1. Introduction

The Ranque Hilsch Vortex Tube (RHVT) is a device that generates cold and hot gas at two outlets from compressed gas at the inlet, as shown in figure 1. When high-pressure gas enters tangentially into the vortex chamber via the inlet nozzle, a swirling flow is generated inside the vortex chamber by a vortex generator. When the gas swirls to the centre of the chamber, it is expanded and cooled. In the vortex chamber, one part of the rotating gas moves towards the hot end, and another part exits via the cold exhaust. Fraction of the gas moving toward the hot end of the vortex tube reverses

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and moves axially from the hot end to the cold end. At the hot exhaust, the gas escapes with a higher temperature, while at the cold exhaust, the gas has a lower temperature compared to the inlet temperature.

**Fig. 1.** Components of a RHVT and its inlet and outlet streams and side view; (Inset: a vortex generator)

RHVT is a simple and compact low cost device. As this device has no moving part it is maintenance free. The device does not require any power supply and the temperature of hot and cold end fluid can be easily adjusted by changing the opening of the hot end control valve (shown in Fig. 1). Apart from a thermal separator, RHVT is also mentioned in literature as a gas species separation device, but not many work on gas species separation with the RHVT has been reported in the literature.

Linderstrom-Lang (1964, 1966) observed that a RHVT of shorter length produced a lower thermal separation and higher component separation. Marshall (1977) used several different gas mixtures in a variety of sizes of vortex tubes and confirmed the effect of the gas separation as described by Linderstrom-Lang (1966). A critical inlet Reynolds number was identified at which the separation was a maximum. Cockrell (1998) have reported use of vortex tubes for gas liquefaction and mixture separation. Kulkarni and Sardesai (2002) used a vortex tube for separating methane and nitrogen from a mixture and found that there was partial gas separation which resulted into higher concentration of methane at one exit in comparison to the inlet and a lower concentration at the other exit. Mohammadi and Farhadi (2014) have investigated the separation performances of a RHVT for a hydrocarbon mixture. They have made a comparison between a distillation column and a cascade made of RHVTs providing similar separation.

Farouk et al. (2009) used an axi-symmetric Computational Fluid Dynamics (CFD) model with Large Eddy Simulation (LES) technique to predict the temperature separation and the separation of nitrogen-helium mixture in a RHVT. The species separation was attributed to the Soret diffusion. Presence of radial separation in a strong centrifugal field was also suggested by the authors. Dutta et al. (2011) have used a three dimensional CFD model to investigate energy and species separation in a RHVT with compressed air at normal room temperature and cryogenic temperature. Air was used as the working fluid. Rafiee and Sadeghiazad (2016) carried out parametric optimization for separation performance using experimental methods and 3D-CFD simulation. Manimaran (2016) had calculated energy separation by simulating a three dimensional flow field in Ranque-Hilsch vortex tube. Thakare et al. (2015) have carried out numerical investigation of flow characteristics in a vortex tube.
The theoretical prediction of the RHVT performance requires the aid of CFD model. The modeling involves discretisation of flow domain and solution of high speed compressible flow field. Simulation of flow field and temperature distribution requires solution of three dimensional Navier Stokes equations along with the energy equation and ideal gas assumption. Generation of grid and running the solver requires large computation time. The computation time overhead increases considerably while employing advanced computational tools like Reynolds Average Navier Stokes (RANS), LES, Direct Numerical Simulation (DNS). In the present work a simple one dimensional mathematical model has been developed. This model can compute mass transfer in a counter current RHVT based on inlet pressure, temperature, concentration, flow rate and outlet pressure, temperature and flow rate. This model can be used for rapid design and simulation of species separation in a RHVT. The model considers simultaneous heat and mass transfer in a RHVT. The well established Chilton Colburn analogy is used under acceptable domain to determine the mass transfer coefficient based on the heat transfer coefficient.

Several research papers and book chapters had been published on the topic of heat, mass, and momentum transfer analogies for the fully developed turbulent as well as laminar flow of fluids in circular tubes. As the flow inside RHVT is rotational in nature, the literature survey carried out is limited to heat and mass transfer analogy applied to rotational flow.

For turbulent heat transfer from a rotating disk, semi-empirical calculations based on the friction analogy were first carried out by Cobb and Saunders (1956) and later refined by Kreith et al. (1959). The analogy results of Kreith et al. (1959) agree favourably with the experimental data. Tien and Campbell (1963) experimentally investigated convective heat transfer from isothermal rotating cones by measuring sublimation rate from naphthalene-coated cones and using the analogy between heat and mass transfer. Dunthorn (1968) had used the Chilton Colburn analogy to design a batch desublimer. Kyung et. al. (2007) determined the detailed heat transfer coefficients using a heat and mass transfer analogy in order to study, the effects of bleed flow on heat/mass transfer in a rotating smooth square channel. Wilk (2011) has applied the mass/heat transfer analogy to the investigation of convective laminar heat transfer in rotating and stationary short mini-channels. Wilk (2011) has provided a general form of the mass/heat transfer analogy, assumptions and the dimensionless numbers and equations describing the analogy. He has described the application of the mass/heat transfer analogy by Chilton and Colburn (1934) in the study of heat transfer in short rotating mini-channels. An important conclusion made by the author was that the uncertainty in mass/heat transfer analogy is of the same order of magnitude as the uncertainty of the heat transfer coefficient determined by means of a specific measuring technique or some suitable correlation. Harmand et. al. (2013) have reviewed methodologies which are used to study rotating and stationary surfaces with or without jet impingement and rotor-stator configurations with or without jet. The general experimental technique of naphthalene sublimation has been used by researchers like Chen (1998), Cho (2001, 2003), He (2005) and others to study the local convective heat transfer in rotating disk configurations. The Reynolds analogy is then used to link the mass transfer to the heat transfer using the local Nusselt, Sherwood, Schmidt and Prandtl numbers. Venkatesan and Fogler (2004) have mentioned that in processes, such as wax deposition in petroleum pipelines and frost formation on heat exchanger surfaces, where the temperature gradient creates a concentration gradient, the usual heat-mass transfer analogy is not valid. In the present work the concentration gradient is a result of radial pressure gradient due to a strong centrifugal field.
In the present work a set of experiments were carried out with three vortex generators of different capacities in order to validate the mathematical model. Experimental data involving parameters like inlet and outlet volumetric flow rates, temperature and pressure of gas mixture at the compressed air inlet and outlet of the Ranque Hilsch vortex tube, were generated. Air, considered as a binary mixture of nitrogen and oxygen gas was used for experimental purpose. Gas compositions at compressed air inlet, cold outlet and hot outlet of the RHVT were measured. The separation factors thus obtained experimentally are compared with their values computed from the mathematical model. The comparative values are presented in the results and discussion section.

2. Experimental Setup

The schematic diagram of the RHVT test facility is shown in figure 2. The RHVT was thermally insulated and three RTDs $T_1$, $T_2$ and $T_3$ were connected to the inlet, cold outlet and hot outlet respectively for measuring the gas temperature. Three more RTDs $T_4$, $T_5$ and $T_6$ were connected to the horizontal tube of the RHVT at equal intervals to measure the skin temperature. Both the cold and hot outlets were connected with two horizontal cylindrical chambers in order to obtain
mixing cup temperatures at the outlets. Three dial gauges were connected to the inlet, cold outlet and hot outlet respectively for measuring the gas pressure. Compressed air from the air compressor passed through a dehumidifier and became moisture free. This high pressure air entered the RHVT inlet through a rotameter and the inlet volumetric flow rate was measured from the rotameter reading. Similarly the volumetric flow rates of air coming out from the hot and cold outlets of the RHVT were measured from the two rotameters attached to these outlets respectively. Pressure and volumetric flow rate of air entering the RHVT were controlled by a control valve fitted in the inlet line. Two sample collection lines were connected to the hot and cold ends of the RHVT in order to collect gaseous sample for mass spectrometric analysis. Mass spectrometric analysis of the samples was carried out using a HIDEN make Residual Gas Analyzer (RGA) mass spectrometer. Details of the experimental setup is described in Chatterjee et al. (2016).

All the measuring instruments are locally procured and conform to international standard. PT100 temperature sensors were used to measure the gas temperatures. Temperatures are logged in a RTD scanner with accuracy of 0.1°C. The maximum possible error in the case of temperature measurement was calculated from the minimum values of the temperatures measured and the accuracy of the instrument. The error in the temperature measurement is:

$$\frac{\partial T}{T} = \sqrt{\left(\frac{\partial T_{RTD}}{T_{min}}\right)^2 + \left(\frac{\partial T_{scanner}}{T_{min}}\right)^2} = \sqrt{\left(\frac{0.5}{10}\right)^2 + \left(\frac{0.1}{10}\right)^2} = 0.05 = 5\%$$

(1)

Bourdon-tube-type dial gauges were used to measure the gas pressure. These gauges conform to IS 3624:1987 (R2004) standards. The error in the transducer and reading error are of the order 0.01 kg/cm². Hence the error in the pressure measurement is:

$$\frac{\partial P}{P} = \sqrt{\left(\frac{\partial P_{tran}}{P_{min}}\right)^2 + \left(\frac{\partial P_{reading}}{P_{min}}\right)^2} = \sqrt{\left(\frac{0.01}{1.5}\right)^2 + \left(\frac{0.01}{1.5}\right)^2} = 0.01 = 1\%$$

(2)

Flow measurements were made using gas flow rotameter. Uncertainty analysis conducted according to the standard procedures has shown that the accuracy in the flow rate measurement is +/- 2% of full flow.

**Table 1** Geometrical dimensions of RHVT used in experiments

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube length</td>
<td>0.102 m</td>
</tr>
<tr>
<td>Tube diameter</td>
<td>0.010 m</td>
</tr>
<tr>
<td>Cold end orifice diameter</td>
<td>0.015 m</td>
</tr>
<tr>
<td>Hot end orifice diameter</td>
<td>0.010 m</td>
</tr>
<tr>
<td>Number of nozzles</td>
<td>Six</td>
</tr>
</tbody>
</table>

A FRIGID-X™ make medium size (stainless steel body) vortex tube was used for all the experiments. The dimensions of the vortex tube are given in the table 1 above. Vortex generators of different capacities can be used with this vortex tube interchangeably.
The principal component of the RHVT is the vortex generator, which is an interchangeable, stationary part that can regulate the volume of compressed air, altering the air flows and temperature ranges that can be produced with the vortex tube. The capacity of gas that can be handled by a vortex generator depends on the size and number of the angular slots engraved into it, while the direction of the air flow depends on the angle of the slots. Three different FRIGID-X™ make vortex generators namely 10H, 25H and 40H have been used for experimentation. The specification of the vortex generators are given in table 2.

### Table 2 Specification of vortex generators used for experiments

<table>
<thead>
<tr>
<th>Model</th>
<th>Model no.</th>
<th>Inlet flow rate (m³/s)</th>
<th>Inlet pressure (Pa)</th>
<th>Cooling capacity (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10H</td>
<td>50010H</td>
<td>4.71947 x 10⁻³</td>
<td>6.9 x 10⁵</td>
<td>214</td>
</tr>
<tr>
<td>25H</td>
<td>50025H</td>
<td>11.7987 x 10⁻³</td>
<td>6.9 x 10⁵</td>
<td>527</td>
</tr>
<tr>
<td>40H</td>
<td>50040H</td>
<td>18.8779 x 10⁻³</td>
<td>6.9 x 10⁵</td>
<td>849</td>
</tr>
</tbody>
</table>

### 3. One Dimensional Mathematical Model

When a binary gas mixture of a heavier species with a lighter species enters the vortex chamber of the RHVT a vortex flow is generated which advances towards the hot end of the tube. In the present model the swirling flow path is divided into two distinct flow streams. The first is the peripheral stream (indicated by numeral 2 in the figure 3) that moves towards the hot end along the z direction, located far from the centre line and adjacent to the cylindrical tube wall, while the second is the axial stream that moves in parallel to the peripheral stream (indicated by numeral 3 in figure 3) in the same direction but located near the centre line. A centrifugal separation field is created due to high speed rotation of the gas mixture which acts on the atoms/molecules of different masses in the gas mixture. Description of different parts of the present model as shown in figure 3 is given in table 3.
Table 3 Description of different components of the RHVT model

<table>
<thead>
<tr>
<th>Component No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Inlet</td>
</tr>
<tr>
<td>2</td>
<td>Peripheral stream, enriched in heavier species</td>
</tr>
<tr>
<td>3</td>
<td>Axial stream, depleted in heavier species</td>
</tr>
<tr>
<td>4</td>
<td>Peripheral stream out; hot end</td>
</tr>
<tr>
<td>5</td>
<td>Axial stream out; cold end</td>
</tr>
<tr>
<td>6</td>
<td>Transfer of heavier species from axial to peripheral stream</td>
</tr>
<tr>
<td>7</td>
<td>Axial return stream, depleted in heavier species</td>
</tr>
<tr>
<td>8</td>
<td>Hot end control valve</td>
</tr>
<tr>
<td>9</td>
<td>Centre line</td>
</tr>
<tr>
<td>10</td>
<td>Change in flow direction of axial stream</td>
</tr>
</tbody>
</table>

As this two parallel streams move towards the hot end atoms/molecules of the heavier species experience higher centrifugal force towards the periphery as compared to the lighter species and preferentially get transferred to the peripheral stream. Thus along the length of the vortex tube from the gas inlet to the hot outlet the peripheral stream gets enriched in heavier species while the axial stream gets depleted in heavier species. Consequently for a binary mixture the axial stream gets enriched in lighter species and the peripheral stream gets depleted in lighter species.

In the present model it has been assumed that the peripheral stream is withdrawn from the peripheral opening of the hot end which is controlled by the hot end control valve. Hence the flow rate of the peripheral stream is experimentally measured by flow meter connected to the hot end of the vortex tube. On the other hand the axial stream that moves in parallel to the peripheral stream up to the hot end control valve, collides with the solid wall of the valve and turns back in the opposite direction and a new counter flow stream is generated which moves in the opposite direction of the axial stream, along the centre of the vortex tube. This stream is indicated by numeral 7 in the figure 3 above and is designated as axial return stream. It is assumed in the present model that while the mass transfer takes place only between peripheral and axial stream, the axial return stream does not take part in the process of mass transfer and hence it’s concentration does not change from the hot end to the cold end. The axial return stream comes out from the cold side outlet and the flow rate is measured by flow meter connected to the cold end of the vortex tube.

![Fig. 4. A control volume to develop the mass transfer model](image-url)
Figure 4 shows a control volume of thickness $\Delta z$, cross section area $A$ and perimeter $P_e$ of the tube portion of the RHVT. Let the gas enters the control volume from left side with a velocity $v$, and a concentration of heavier species $x_z$. Gas comes out from the right side of the control volume with a concentration of heavier species $x_{z+\Delta z}$. Therefore we get the rate of change of concentration of heavier species stored inside the control volume per unit volume

$$\frac{\partial}{\partial t} \left( \frac{x \Delta z}{A \Delta z} \right) = \frac{\partial x}{\partial t}$$  \hspace{1cm} (3)

Now the volumetric flow rate of heavier species can be expressed by,

$$Q = v(x_z - x_{z+\Delta z}) A$$  \hspace{1cm} (4)

Hence, Differential representation of the change in concentration of heavier species in terms of spatial gradient of the heavier species flux is given by

$$\frac{Q}{A \Delta z} = -v \frac{\partial x}{\partial z}$$  \hspace{1cm} (5)

Now, if $m$ is the mass of heavier species reaching from the axial stream to a unit area of the peripheral wall then total moles of heavier species reaching the wall of the control volume

$$H_{wall} = P_e \Delta z \frac{m}{M_1}$$  \hspace{1cm} (6)

Therefore, rate of molar flux of heavier species reaching the peripheral wall of the control volume, per unit volume

$$\frac{\partial}{\partial t} \left( \frac{H_{wall}}{A \Delta Z} \right) = \frac{P_e}{M_1 A} \frac{\partial m}{\partial t}$$  \hspace{1cm} (7)

Hence from equation (3) to (7), a component balance along the control volume gives

$$\frac{1}{v} \frac{\partial x}{\partial t} = -\frac{\partial x}{\partial z} - \frac{P_e}{M_1 v A} \left( \frac{\partial m}{\partial t} \right)$$  \hspace{1cm} (8)

Now, the term $vA$ in equation (8) is the total volumetric flow rate of the gas mixture. Hence if $V$ is the flow rate of the lighter species then

$$V = vA(1 - x)$$  \hspace{1cm} (9)

Hence from equation (8) and (9) we get

$$\frac{1}{v} \frac{\partial x}{\partial t} + \frac{\partial x}{\partial z} + \frac{(1-x)}{V} \frac{P_e}{M_1} \left( \frac{\partial m}{\partial t} \right) = 0$$  \hspace{1cm} (10)

The gas concentration at a point does not change rapidly with time, hence neglecting the first term in equation (10) we can write the differential equation for mass transfer as
\[
\frac{dx}{dz} = -\frac{(1-x) P_e}{V M_1} \left( \frac{\partial m}{\partial t} \right)_z \quad (11)
\]

Now, at any point within the RHVT at a distance \(z\) from the inlet, the rate of mass transfer to the peripheral region from the core region may be expressed as

\[
\left( \frac{\partial m}{\partial t} \right)_z = M_1 K (P - P_w) = M_1 KP(1 - \beta) \quad (12)
\]

The mass transfer coefficient is difficult to obtain for different systems. Therefore we can apply Chilton Colburn analogy [Dunthorn, 1968] within its applicable range (i.e. inside the flow domain \(Re > 10000\) and \(0.7 < Pr < 160\)). Hence we can get the mass transfer coefficient from the following equation in terms of heat transfer coefficient.

\[
j_m = \frac{K P_g f S c^2}{(G/M_m)} = h \frac{Pr^{2/3}}{c_m G} = j_h \quad (13)
\]

It is assumed that the flow is fully developed at the downstream of the vortex generator which is also a necessary condition for application of Chilton Colburn analogy. Mass transfer coefficient can be calculated from heat transfer coefficient using equation 13. The accuracy of the mass transfer coefficient thus estimated depends on the accuracy of the heat transfer coefficient data. The heat transfer coefficient data could be either experimental or the one obtained from a suitable correlation. Thus the mass transfer coefficient so obtained is as much accurate as the heat transfer coefficient, as stated by Wilk (2011).

The term, \(P_g f\), in the above relationship on expansion gives

\[
P_{gf} = (P_i - P_{iw})/\ln(P_i - P_{iw}) \quad (14)
\]

where,

\[
P_i = P_T - P = P_T - xP_T \quad (15)
\]

and,

\[
P_{iw} = P_T - P_w \quad (16)
\]

Thus,

\[
P_{gf} = P(1 - \beta)/\ln\{(1 - P_w/P_T)/(1 - x)\} \quad (17)
\]

Hence combining equations (12), (13) and (17) we get

\[
\left( \frac{\partial m}{\partial t} \right)_z = \frac{M_1 h N \ln\{(1-P_w/P_T)/(1-x)\}}{M_mC_m} \quad (18)
\]

where,

\[
N = (Pr/Sc)^{2/3} \quad (19)
\]

Now from equation (11) and (18) we can get the ordinary differential equation for mass transfer as

\[
\frac{dx}{dz} = -\frac{(1-x) P_e N}{V M_1} \ln\{(1 - P_w/P_T)/(1 - x)\} \quad (20)
\]
The heat transfer coefficient $h$ in equation (20) can be calculated using either experimental data or some empirical correlation. One such empirical correlation is Seider-Tate correlation that is applicable for forced convection for turbulent pipe flow under the condition that $0.7 < Pr < 160$, $Re > 10000$ and $\frac{L}{D} \geq 10$. In the present calculation this correlation is used to determine the heat transfer coefficient and it is given by

$$Nu = 0.027 Re^{4/5} Pr^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$$

(21)

The Sieder-Tate correlation takes into account the change in viscosity $\mu$ and $\mu_s$, due to change in temperature between the gas average temperature at the core region and the heat transfer surface temperature, respectively.

In the present work the cold ($T_2$) and hot ($T_3$) outlet temperature has been experimentally determined. A linear variation of temperature of the peripheral stream is assumed in the mathematical model from inlet temperature (ambient temperature) up to the hot outlet temperature ($T_3$). Similarly the temperature of the axial stream also varies linearly from inlet ambient temperature to a temperature equal to the cold outlet temperature ($T_2$) near the hot end control valve. The axial return stream is maintained at this temperature ($T_2$) throughout the length of the RHVT i.e. the stream remains at isothermal condition.

4. Results and Discussions

Three sets of experimental results were compared with the result predicted by the present one dimensional mathematical model which calculates mass transfer inside the vortex tube. For each set of experiments a medium size RHVT fitted with a different capacity vortex generator had been used. The model numbers of those vortex generators were 10H, 25H and 40H. The technical specifications of these vortex generators were given in section 3. In each set of experiments hot end control valve opening was varied from 0.5 opening to 5.0 opening in steps of 0.5. Here 1.0 opening of the valve means 360° open from fully closed condition. For each opening of the hot end control valve the inlet pressure was varied in eight levels. These are 147.0, 196.0, 245.0, 294.0, 313.0, 392.0, 441.10 and 490.20 kPa respectively. For each combination of hot end control valve opening and inlet pressure, only stabilized values of volumetric flow rate, pressure and temperature of gas mixture at inlet and two outlets (cold and hot) were measured. Simultaneously samples of gas mixture were collected from both the outlets and their compositions were determined as already described in section 2.

Air, considered as a binary mixture of nitrogen and oxygen, was taken as the working fluid. The composition of air was given in percentage of the heavier species (i.e. oxygen) concentration. The oxygen sensor used (RGA quadrupole mass spectrometer) responds to the absolute concentration of oxygen in air. Thus with changes in pressure and temperature, the output of the sensor changes, even though the relative concentration of oxygen has not changed. The sensor’s responses to pressure and temperature are repeatable, thus it is possible to measure pressure and temperature and correct for them in software.
4.1 Feed flow rate vs. separation factor

The separation factor of the lighter species for a binary separation is defined by

\[ \alpha = \frac{1-N_3}{N_3} \frac{N_2}{1-N_2} \]  

(22)

A computer code based on the mathematical model developed can predict the variation of compositions of the two parallel streams (axial and peripheral) along the length of the RHVT. Hence we can obtain from the computer code the composition of two outlet streams from the RHVT. Using equation (22) the binary separation factors were computed from the code and compared with the corresponding experimental values. The computer code requires experimentally measured temperature and pressure data at the inlet and two outlets. Hence separation factors were computed at experimental conditions where temperature and pressure data were available. Only stable data values which do not change with time were considered. The experimental conditions, where flow rate or pressure values showed instability were not considered. The oxygen concentration values at the cold and hot outlets were computed using the computer code. Corresponding values of separation factors were plotted against the feed flow rate for different values of hot end valve opening, for three different vortex generators.

Species separation was observed in all the three cases shown in figure 5. Comparing the three graphs in figure 5, it was observed that for a fixed value of inlet pressure, feed flow rate increases with increasing capacity of the vortex generators. This is due to increasing conductance across the vortex chamber. The range of values of the separation factors also increase from lower to higher capacity of vortex generators as the higher capacity generators can produce higher swirling motion in the gas mixture. It is observed that separation factor increases continuously with feed flow rate for 10H vortex generator. The maximum value of separation factor for 10H generator might be obtained at a higher value of inlet flow rate which is limited by the available inlet pressure from the compressor in our setup.

The separation factor goes through a maxima for 25H and 40H generators. This phenomenon is due to reduction of difference in partial pressure of the heavier species between peripheral and axial regions. The optimum value of feed flow rate is 0.0175 m³/s for 25H vortex generator while it is 0.025 m³/s for 40H vortex generator.

The cold mass fraction is defined by

\[ \theta_c = \frac{m_c}{m_c + m_h} \]  

(23)

Thus if \( m_{tot} = m_c + m_h \) is the total inlet flow rate then according to the model the flow rate of the axial stream (as well as axial return stream) is given by \( m_{tot} \theta_c \) and the flow rate of the peripheral stream is given by \( m_{tot} (1 - \theta_c) \). Now the values of the opening of hot end control valve is an indication of the cold mass fraction. In the present work values of opening of hot end control valve were considered as a parameter as they are most frequently used for controlling the cold mass fraction by the operator. It can be seen from the graphs in figure 5 b and c that with increasing values of hot end control valve opening, species separation initially increases, goes through a maximum and then comes down. This is similar to optimum value of product to feed ratio for species separation reported in gas centrifugal separation of binary mixture of gases. The optimum value of hot end control valve opening is found as 3.0 turns for both. Hence for this value of hot end control valve opening is found as 3.0 turns for both.
control valve opening and for optimum values of feed flow rates, the maximum value of separation factor obtained were 1.27 and 1.39 for 25H and 40H vortex generator respectively.

**Fig. 5.** Feed flow rate vs. separation factor (computed) at different values of hot end valve opening for vortex generators a. 10H  b. 25H and c. 40H
4.2 Comparison of computational and experimental results

It was seen from the previous section that the mathematical model developed in the present work can predict the species separation performance for a wide range of values of hot end control valve opening and inlet flow rates. For the comparison purpose of computational and experimental results the values of hot end control valve opening selected are 2.5, 3.0, 3.5 and 4.0 turns.

![Graphs showing comparison between computed and experimental separation factor for different hot end valve openings](image)

**Fig. 6.** Feed flow rate vs. experimentally obtained and computed separation factor for different hot end valve opening for 10H vortex generator

It may be noted here that since turns of valve may not be able to reproduce the same opening every time for a large number of experiments, we can replace the values of turns with reading of a micrometer attached with the valve of the hot end opening valve.
Figure 6 shows the comparison of experimental and computed values of separation factors for the 10H vortex generator plotted with inlet feed flow rate. The computed values are shown with a solid line while the experimental values of separation factors are shown with solid squares. It can be seen from the plots that the computed results have predicted the experimental values of separation factor with acceptable accuracy. The experimental values of separation factors are slightly over predicted for 2.5 opening while these values are slightly under predicted for 4.0 opening.

![Graph showing comparison of experimental and computed separation factors for 10H vortex generator](image)

**Fig. 6.** Comparison of experimental and computed separation factors for 10H vortex generator.

Figure 7 shows the comparison of experimental and computed values of separation factors for the 25H vortex generator plotted with inlet feed flow rate. Here also it can be seen from the plots that the computed results have predicted the experimental values of separation factor with acceptable accuracy.

![Graph showing comparison of experimental and computed separation factors for 25H vortex generator](image)

**Fig. 7.** Feed flow rate vs. experimentally obtained and computed separation factor for different hot end valve opening for 25H vortex generator.
the difference between computed values and experimentally obtained values of separation factors are slightly higher for lower values of hot end control valve opening like 2.5 to 3.5. It can be observed that for a higher value of hot end control valve opening like 4.0, the agreement between experimental and theoretical value improves.

Comparison of experimental and computed values of separation factors for the 40H vortex generator is plotted in figure 8. It can be seen from the graphs that for all the different values of hot end control valve opening the differences between experimental and computed values of separation factors are relatively small.
4.3 Predicted variation in gas composition for axial and peripheral streams

The variation of oxygen concentration along the length of the vortex tube from the inlet to the hot outlet in the two streams (peripheral and axial) of the flowing gas mixture for different values of inlet pressure have been plotted in the graphs shown in figure 9, 10 and 11 for vortex generators.

10H, 25H and 40H respectively. The plots are given for all the four cases of hot end control valve openings i.e. 2.5, 3.0, 3.5 and 4.0. No experimental data for variation of oxygen concentration along the peripheral and axial stream are available and can be obtained from a computational fluid dynamics simulation of flow field followed by solution of the species conservation equation.
It can be seen from the results that for 10H generator (figure 9) for a fixed value of the hot end valve opening the concentration difference of heavier species in the gas mixture (i.e. oxygen) between the axial and peripheral streams increases continuously with increasing value of inlet pressure which is directly proportional to inlet feed flow rate. This implies that the optimum separation factor for this generator is achievable at a still higher value of inlet feed flow rate. Further it can be seen from the plots for 25H and 40H vortex generators that the maximum separation of oxygen concentration between the peripheral and axial streams occurs at a value of
inlet pressure which is less than the highest value of inlet pressure considered in this study. Hence it can be concluded that there is an optimum value of inlet pressure at which maximum species separation is observed for a given generator. It can also be concluded from the graphs that this optimum value of inlet pressure also changes with variation in hot end valve opening. This point can be illustrated by comparing the last two sets of graphs in figure 10 for hot end valve opening values of 3.5 and 4.0 respectively. We can see while the optimum value of inlet pressure is 313.14 kPa for the former case, this value is 294.12 kPa for the latter case. Hence we can conclude that species separation depends on the values of inlet pressure and hot end valve opening. Further it can

Fig. 11. Distance vs. oxygen concentration in axial and peripheral streams for different hot end valve opening for a 40H generator.
be seen that while inlet pressure is an indicator of inlet flow rate, hot end valve opening is an indicator of the cold mass fraction of the unit as defined by equation (23).

Along with the concentration build up of heavier species near the peripheral wall of the RHVT, the partial pressure of heavier species in gas stream at wall, \( P_w \), increases gradually. Hence the driving force for mass transfer, i.e. \( P - P_w \), in equation (12), starts decreasing along the length of the RHVT near the hot end. Hence the concentration plots for the heavier species in the peripheral stream gradually flattens for all the hot end valve openings and inlet pressure values as shown in figure 9, 10 and 11.

5. Conclusions

This paper presents results obtained from experiments and a one dimensional mathematical model for mass transfer in a medium sized RHVT using air as a binary mixture of nitrogen and oxygen. The following conclusions can be drawn from the results obtained:

- Using Chilton Colburn analogy under acceptable domain we can calculate the mass transfer coefficient from the heat transfer coefficient for a device like RHVT where mass and heat transfer occurs simultaneously.

- The computed values of separation factors from the one dimensional mathematical model compares well with the experimental values of separation factors for the RHVT.

- In the range of inlet pressure considered while separation factor increases continuously with feed flow rate for 10H vortex generator, its value goes through a maxima for 25H and 40H vortex generators.

- The optimum value of inlet flow rate for which separation factor becomes maximum is found to be a function of hot end valve opening which is an indicator of cold mass fraction.

- The gradient of molar concentration of heavier species near the wall of RHVT initially increases along with the distance from the inlet towards the hot outlet and then flattens gradually near the hot outlet. This phenomenon is observed due to reduction in driving force for mass transfer.

- The present mathematical model can compute mass transfer in a counter current RHVT based on inlet pressure, temperature, concentration, flow rate and outlet pressure, temperature and flow rate. This model can be used for rapid design and simulation of species separation in a RHVT.

- The mathematical model is tested over a range of three different vortex generators (10H, 25H and 40H), hot end valve opening values (1.5 opening to 5.0 opening) and inlet pressure (147 kPa to 392 kPa). The agreement between experimental results and values predicted by the present mathematical model was found acceptable.
The mass transfer coefficient is calculated from heat transfer coefficient using the heat and mass transfer analogy. Therefore an error in the value of the heat transfer coefficient leads to error in the value of the mass transfer coefficient calculated. This is a limitation of the present model.

Another limitation of the present model is that because it is an one dimensional model, it cannot capture the onset as well as effects of turbulence which is three dimensional in nature.

The present model requires experimentally measured thermal profile to predict the concentration profile of a binary mixture of two gas species along the length of the RHVT.

6. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>$A$</td>
<td>Cross section area of tube portion of RHVT (m$^2$)</td>
</tr>
<tr>
<td>$C_m$</td>
<td>Mean specific heat of gas in stream (J/kg·K)</td>
</tr>
<tr>
<td>$D$</td>
<td>Wetted diameter of vortex tube (m)</td>
</tr>
<tr>
<td>$G$</td>
<td>Mass velocity of gas mixture (kg/s)</td>
</tr>
<tr>
<td>$h$</td>
<td>Heat transfer coefficient (W/m$^2$·K)</td>
</tr>
<tr>
<td>$H_{wall}$</td>
<td>Total moles of heavier species reaching the wall of the control volume (kg·mole)</td>
</tr>
<tr>
<td>$j_h$</td>
<td>Heat transfer &quot;j&quot; factor</td>
</tr>
<tr>
<td>$j_m$</td>
<td>Mass transfer &quot;j&quot; factor</td>
</tr>
<tr>
<td>$K$</td>
<td>Mass transfer coefficient (m/s)</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of vortex tube (m)</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of heavier species reaching a unit area of the RHVT wall (kg·mole/s·m$^2$)</td>
</tr>
<tr>
<td>$m_c$</td>
<td>Cold outlet mass flow rate (kg/s)</td>
</tr>
<tr>
<td>$m_h$</td>
<td>Hot outlet mass flow rate (kg/s)</td>
</tr>
<tr>
<td>$m_{tot}$</td>
<td>Total inlet flow rate (kg/s)</td>
</tr>
<tr>
<td>$M_1$</td>
<td>Molecular weight of heavier species (kg·mole)</td>
</tr>
<tr>
<td>$M_m$</td>
<td>Mean molecular weight of gas (kg·mole)</td>
</tr>
<tr>
<td>$N$</td>
<td>$\left(\frac{Pr}{Sc}\right)^{2/3}$</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt Number, $\frac{hD}{K}$</td>
</tr>
<tr>
<td>$N_2$</td>
<td>Mole fraction of heavier species at hot outlet</td>
</tr>
<tr>
<td>$N_3$</td>
<td>Mole fraction of heavier species at cold outlet</td>
</tr>
<tr>
<td>$P$</td>
<td>Partial pressure of heavier species in gas stream at core (Pa)</td>
</tr>
<tr>
<td>$P_e$</td>
<td>Perimeter of flow channel (m)</td>
</tr>
<tr>
<td>$P_i$</td>
<td>Partial pressure of lighter species in gas stream (Pa)</td>
</tr>
<tr>
<td>$P_{nw}$</td>
<td>Partial pressure of lighter species at wall (Pa)</td>
</tr>
<tr>
<td>$P_{gf}$</td>
<td>Logarithmic mean partial pressure of heavier species (Pa)</td>
</tr>
<tr>
<td>$P_{min}$</td>
<td>Experimentally measured minimum value of the pressure (kg/cm$^2$)</td>
</tr>
<tr>
<td>$P_T$</td>
<td>Total pressure (Pa)</td>
</tr>
<tr>
<td>$P_w$</td>
<td>Partial pressure of heavier species in gas stream at wall (Pa)</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number, $\frac{C_m\mu}{K}$</td>
</tr>
</tbody>
</table>
Flow rate of heavier species (kg mole/s)

Reynolds number, \( \frac{\rho_{d} \cdot \nu}{\mu} \)

Schmidt number, \( \frac{\mu}{\rho \cdot \nu} \)

Time (s)

Experimentally measured minimum value of the temperature (°C)

Inlet temperature (K)

Cold outlet temperature (K)

Hot outlet temperature (K)

RHVT skin temperature (K)

Velocity of bulk gas stream (m/s)

Flow rate of lighter species (kg mole/s)

Bulk average mole fraction of heavier species in gas stream

Bulk average mole fraction of heavier species at axial location z

Distance along length of RHVT (m)

Diffusivity

Separation factor

\( \frac{P_{w}}{P} \)

Error in pressure transducer reading (kg/cm²)

Accuracy in pressure transducer (kg/cm²)

Maximum possible error in temperature measurement (°C)

Accuracy in RTD scanner (°C)

Density of the fluid (kg/m³)

Cold mass fraction

Viscosity of gas at core (Pa-s)

Viscosity of gas at wall surface (Pa-s)

Thickness of control volume (m)

7. References


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