A Study on Non-Response for the Estimation of Current Population Ratio in Sampling on Two Occasions

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Abstract

In this article, we attempt the problem of estimation of the population ratio of mean in mail surveys. This problem is conducted for current occasion in the context of sampling on two occasions when there is non-response (i) on both occasions, (ii) only on the first occasion and (iii) only on the second occasion. We obtain the loss in precision of all the estimators with respect to the estimator of the population ratio of mean when there is no non-response. We derive the sample sizes and the saving in cost for all the estimators, which have the same precision than the estimator of the population ratio of mean when there is no non-response. An empirical study that allows us to investigate the performance of the proposed strategy is carried out.

Keywords: Successive Sampling; Non-response; Estimator of the Ratio; Loss in Precision

1. Introduction

Jessen (1942), Tikkiwal (1951), Yates (1949), Patterson (1950), Eckler (1955) and Raj (1968) contributed towards the development of the theory of unbiased estimation of mean of characteristics in successive sampling. In many practical situations the estimate of the population ratio and product of two characters for the most recent occasion may be of considerable interest. The theory of estimation of the population ratio of two characters over two occasions has been considered by Rao (1957), Rao and Pereira (1968), Okafor and Arnab (1987), Okafor (1992), Artés and García (2001), García and Artés (2002) among others. Further, García (2008) presented some sampling strategies for estimating, by a linear estimate, the population product of two characters over two occasions.

Hansen and Hurwitz (1946) suggested a technique for handling the non-response in mail surveys. These surveys have the advantage that the data can be collected in a relatively inexpensive way.

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Okafor (2001) extended these surveys to the estimation of the population total in element sampling on two successive occasions. Later, Choudhary et al. (2004) used the Hansen and Hurwitz (HH) technique to estimate the population mean for current occasion in the context of sampling on two occasions when there is non-response on both occasions. More recently, Singh and Kumar (2010) used the HH technique to estimate the population product for current occasion in the context of sampling on two occasions when there is non-response on both occasions and García and Oña (2011) used the HH technique to estimate the change of mean and the sum of mean for current occasion in the context of sampling on two occasions when there is non-response on both occasions. However, non-response is a common problem with mail surveys: Singh and Kumar, 2011; Kumar et al., 2011; Singh et al., 2011; Kumar, 2012; and Singh, G. N. and Karna, J. P., 2012.

Also, Cochran (1977) and Okafor and Lee (2000) extended the HH technique to the case when the information on the characteristic under study is also available on auxiliary characteristic.

In this article, we develop the HH technique to estimate the population ratio of mean for current occasion in the context of sampling on two occasions when there is non-response (i) on both occasions, (ii) only on the first occasion and (iii) only on the second occasion. An empirical study that allows us to investigate the performance of the proposed strategy is carried out.

2. The Technique

Consider a finite population of \( N \) identifiable units. Let \( (x_i, y_i) \) be, for \( i = 1,2, \ldots, N \), the values of the characteristic on the first and second occasions, respectively. We assume that the population can be divided into two classes, those who respond at the first attempt and those who not. Let the sizes of these two classes be \( N_1 \) and \( N_2 \), respectively. Let on the first occasion, schedules through mail are sent to \( n \) units selected by simple random sampling. On the second occasion, a simple random sample of \( m = np \) units, for \( 0 < p < 1 \), is retained while an independent sample of \( u = nq = n - m \) units, for \( q = 1 - p \), is selected (unmatched with the first occasion). We assume that in the unmatched portion of the sample on two occasions, \( u_1 \) units respond and \( u_2 \) units do not. Similarly, in the matched portion \( m_1 \) units respond and \( m_2 \) units do not.

Let \( m_{\text{hh2}} \) denotes the size of the subsample drawn from the non-response class from the matched portion of the sample on the two occasions for collecting information through personal interview. Similarly, denote by \( u_{\text{hh2}} \) the size of the subsample drawn from the non-response class in the unmatched portion of the sample on the two occasions.

Also, let \( \sigma_{x_j}^2, \sigma_{y_j}^2; j = 1,2 \) and \( \sigma_{x_j(2)}^2, \sigma_{y_j(2)}^2; j = 1,2 \) denote the population variance and population variance pertaining to the non-response class, respectively.

In addition, let \( \bar{x}_{1m}, \bar{y}_{1m}, \bar{x}_{1u} \) and \( \bar{y}_{1u} \) denote the estimator for matched and unmatched portions of the sample on the first occasion, respectively. Let the corresponding estimator for the second occasion be denoted by \( \bar{x}_{2m}, \bar{y}_{2m}, \bar{x}_{2u} \) and \( \bar{y}_{2u} \). Thus, have the following setup:

\[
x_i \ (y_i), \text{ the variable } x \ (y) \text{ on } i \text{th occasion, } i = 1,2,
\]
\[ R_1 = \frac{\bar{y}_1}{\bar{x}_1} (R_2 = \frac{\bar{y}_2}{\bar{x}_2}), \text{ the population ratio on the first (second) occasion,} \]

\[ \hat{R}_1 = \frac{\bar{y}_1}{\bar{x}_1} (\hat{R}_2 = \frac{\bar{y}_2}{\bar{x}_2}), \text{ the estimator of the population ratio on the first (second) occasion,} \]

\[ \hat{R}_{1m} = \frac{\bar{y}_{1m}}{\bar{x}_{1m}} (\hat{R}_{2m} = \frac{\bar{y}_{2m}}{\bar{x}_{2m}}), \text{ the estimator of the population ratio on the first (second) occasion based on the matched sample of } m \text{ units,} \]

\[ \hat{R}_{1u} = \frac{\bar{y}_{1u}}{\bar{x}_{1u}} (\hat{R}_{2u} = \frac{\bar{y}_{2u}}{\bar{x}_{2u}}), \text{ the estimator of the population ratio on the first (second) occasion based on the unmatched sample of } u \text{ units.} \]

\[ \rho_1 (\rho_2), \text{ the correlation coefficients between the variables } y_1 \text{ and } x_1 (y_2 \text{ and } x_2), \]

\[ \rho_3 (\rho_4), \text{ the correlation coefficients between the variables } y_2 \text{ and } x_1 (y_1 \text{ and } x_2), \]

\[ \rho_5 (\rho_6), \text{ the correlation coefficients between the variables } x_1 \text{ and } x_2 (y_1 \text{ and } y_2), \]

\[ \rho_{1(2)} (\rho_{2(2)}), \text{ the correlation coefficients between the variables } y_{1(2)} \text{ and } x_{1(2)} (y_{2(2)} \text{ and } x_{2(2)}), \]

\[ \rho_{3(2)} (\rho_{4(2)}), \text{ the correlation coefficients between the variables } y_{2(2)} \text{ and } x_{1(2)} (y_{1(2)} \text{ and } x_{2(2)}), \]

\[ \rho_{5(2)} (\rho_{6(2)}), \text{ the correlation coefficients between the variables } x_{1(2)} \text{ and } x_{2(2)} (y_{1(2)} \text{ and } y_{2(2)}). \]

<table>
<thead>
<tr>
<th>1st occasion →</th>
<th>( \hat{R}_{1u} )</th>
<th>( \hat{R}_{1m} )</th>
<th>( \hat{R}_{2m} )</th>
<th>( \hat{R}_{2u} )</th>
</tr>
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<tbody>
<tr>
<td>2nd occasion →</td>
<td>( \hat{R}_{2u} )</td>
<td>( \hat{R}_{2m} )</td>
<td>( \hat{R}_{1m} )</td>
<td>( \hat{R}_{1u} )</td>
</tr>
</tbody>
</table>

where

\[ \bar{y}_{1m}^* = \frac{m_1\bar{y}_{1m1} + m_2\bar{y}_{1m2}}{m} \]

Hansen and Hurwitz (1946) estimator for the population mean \( \bar{y}_1 \) on the first occasion for matched portion of the sample.

\[ \bar{y}_{2m}^* = \frac{m_1\bar{y}_{2m1} + m_2\bar{y}_{2m2}}{m} \]

Hansen and Hurwitz (1946) estimator for the population mean \( \bar{y}_2 \) on the second occasion for matched portion of the sample.

\[ \bar{y}_{1u}^* = \frac{u_1\bar{y}_{1u1} + u_2\bar{y}_{1u2}}{u} \]

Hansen and Hurwitz (1946) estimator for the population mean \( \bar{y}_1 \) on the first occasion for unmatched portion of the sample.

\[ \bar{y}_{2u}^* = \frac{u_1\bar{y}_{2u1} + u_2\bar{y}_{2u2}}{u} \]

Hansen and Hurwitz (1946) estimator for the population mean \( \bar{y}_2 \) on the second occasion for unmatched portion of the sample.

\[ \bar{x}_{1m}^* = \frac{m_1\bar{x}_{1m1} + m_2\bar{x}_{1m2}}{m} \]

Hansen and Hurwitz (1946) estimator for the population mean \( \bar{x}_1 \) on the first occasion for matched portion of the sample.
Hansen and Hurwitz (1946) estimator for the population mean $\bar{X}_2$ on the second occasion for matched portion of the sample.

$$\bar{x}_{2m}^* = \frac{m_1\bar{x}_{2m1} + m_2\bar{x}_{2m2}}{m}$$

Hansen and Hurwitz (1946) estimator for the population mean $\bar{X}_1$ on the first occasion for unmatched portion of the sample.

$$\bar{x}_{1u}^* = \frac{u_1\bar{x}_{1u1} + u_2\bar{x}_{1u2}}{u}$$

Hansen and Hurwitz (1946) estimator for the population mean $\bar{X}_2$ on the second occasion for unmatched portion of the sample.

$$\bar{x}_{2u}^* = \frac{u_1\bar{x}_{2u1} + u_2\bar{x}_{2u2}}{u}$$

The procedure of obtaining Hansen-Hurwitz estimators can be seen in Singh and Kumar 2010, p. 978. Also, it can be easily seen that (see Singh and Kumar 2010, p. 979)

$$\text{Cov}(\hat{R}^*_1u, \hat{R}^*_1m) = \text{Cov}(\hat{R}^*_1u, \hat{R}^*_2m) = \text{Cov}(\hat{R}^*_1u, \hat{R}^*_2u) = \text{Cov}(\hat{R}^*_1m, \hat{R}^*_2u) = \text{Cov}(\hat{R}^*_2m, \hat{R}^*_2u) = 0.$$
\[ V(R_2^*) = a^2 \left( \frac{1}{q} + \frac{1}{p} \right) \frac{1}{n \bar{x}_1^2} D^* + c^2 \frac{1}{p n \bar{x}_2^2} E^* + (1 - c)^2 \frac{1}{q n \bar{x}_2^2} E^* - 2ac \frac{1}{p n \bar{x}_1 \bar{x}_2} C^* \]

where

\[ D^* = \{ A + W_2(k - 1)A(2) \}, \quad E^* = \{ B + W_2(k - 1)B(2) \} \]

\[ W_2 = N_2/N; \quad k = m_2/m_{h_2} = u_2/u_{h_2} \]

\[ A = S_{y_1}^2 + R_1^2S_{x_1}^2 - 2R_1 Cov(y_1, x_1); \quad A(2) = S_{y_1(2)}^2 + R_1^2S_{x_1(2)}^2 - 2R_1 Cov(y_1, x_1(2)) \]

\[ B = S_{y_2}^2 + R_2^2S_{x_2}^2 - 2R_2 Cov(y_2, x_2); \quad B(2) = S_{y_2(2)}^2 + R_2^2S_{x_2(2)}^2 - 2R_2 Cov(y_2, x_2(2)) \]

\[ C^* = [ Cov(y_1, y_2) - R_1 Cov(y_2, x_1) - R_2 Cov(y_1, x_2) + R_1R_2 Cov(x_1, x_2)] + \]

\[ W_2(k - 1)[ Cov(y_1, y_2)(2) - R_1 Cov(y_2, x_1)(2) - R_2 Cov(y_1, x_2)(2) + R_1R_2 Cov(x_1, x_2)(2)] \]

The values of \( a \) and \( c \) can be chosen to minimize \( V(R_2^*) \). Equating to zero the derivatives of \( V(R_2^*) \) with respect to \( a \) and \( c \), it follows that the optimum values are

\[ a_{opt} = \frac{pq \bar{x}_1 E^* C^*}{\bar{x}_2(D^* + q^2C^* - q^2C^*^2)} \quad \text{and} \quad c_{opt} = \frac{pD^* E^*}{D^* + q^2C^* - q^2C^*^2} \]

Thus, the estimate with optimum values for \( a \) and \( c \) may be written

\[ R_2^{**} = \frac{pq \bar{x}_1 E^* C^*}{\bar{x}_2(D^* - q^2C^* - q^2C^*^2)} (R_{1u}^* - R_{1m}^*) + \frac{pD^* E^*}{D^* - q^2C^* - q^2C^*^2} R_{2m}^* + \left( 1 - \frac{pD^* E^*}{D^* - q^2C^* - q^2C^*^2} \right) R_{2u}^* \]

and its variance is

\[ V(R_2^{**}) = \frac{E^*}{\bar{x}_2^2 n} \frac{D^* - q^2C^* - q^2C^*^2}{D^* - q^2C^* - q^2C^*^2} \]

Note that if \( q = 0, p = 1 \), complete matching or \( p = 0, q = 1 \), no matching this variance Eq. (2) has the same value,

\[ V(R_2^{**}) = \frac{E^*}{\bar{x}_2^2 n} \]

Thus, for current estimates, equal precision is obtained either by keeping the same sample or by changing it on every occasion. If \( X_1 = \bar{X}_2 \), the estimate give by Eq. (1) is somewhat simplified

\[ R_2^{**} = \frac{pq \bar{x}_1 E^* C^*}{\bar{x}_2(D^* - q^2C^* - q^2C^*^2)} (R_{1u}^* - R_{1m}^*) + \frac{pD^* E^*}{D^* - q^2C^* - q^2C^*^2} R_{2m}^* + \left( 1 - \frac{pD^* E^*}{D^* - q^2C^* - q^2C^*^2} \right) R_{2u}^* \]

but its variance is unchanged, that is,

\[ V(R_2^{**}) = \frac{E^*}{\bar{x}_2^2 n} \frac{D^* - q^2C^* - q^2C^*^2}{D^* - q^2C^* - q^2C^*^2} \]

while if \( W_2 = 0 \), i.e., there is non-response, the \( V(R_2^{**}) \) reduces to

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\[ V(\hat{R}_2) = \frac{B}{X_2^2n} \frac{AB - qk_1^2}{AB - q^2k_1^2} \]

where \( \hat{R}_2 \) is the usual estimator of the ratio of mean for the current occasion in the context of sampling on two occasions when there is complete response, that is, \( \hat{R}_2 = a \hat{R}_{1u} + b \hat{R}_{1m} + c \hat{R}_{2m} + d \hat{R}_{2u} \).

Similarly an estimate of the first occasion is

\[ \hat{R}_1^{**} = \frac{pqX_2E^*C^*}{X_2(D^*E^* - q^2C^2)} (\hat{R}_{2u}^* - \hat{R}_{2m}^*) + \frac{pD^*E^*}{D^*E^* - q^2C^2} \hat{R}_{1m}^* + \left(1 - \frac{pD^*E^*}{D^*E^* - q^2C^2}\right) \hat{R}_{1u}^* \]

Its variance is

\[ V(\hat{R}_1^{**}) = \frac{D^*D^*E^*E^* - q^2C^2}{X_2^2n D^*E^* + \sqrt{D^*E^*E^* - q^2C^2D^*E^*}} \]

Equating to zero the derivative of \( V(\hat{R}_2^{**}) \) with respect to \( q \), we find that the variance \( V(\hat{R}_2^{**}) \) will have its minimum value if we choose

\[ q_{opt}^{(0)} = \frac{D^*E^*E^* - \sqrt{D^*E^*E^* - C^2D^*E^*}}{C^2} \]  \hspace{1cm} (3)

and

\[ V_{min}(\hat{R}_2^{**}) = \frac{E^* D^*E^*E^*E^* - \sqrt{D^*E^*E^* - C^2D^*E^*}}{2D^*E^*} \]

However, if only the estimate using information gathered on the second occasion is considered, the estimator of the population ratio is

\[ \hat{R}^* = p\hat{R}_{2m}^* + q\hat{R}_{2u}^* \]

and its variance is

\[ V(\hat{R}^*) = \frac{E^*}{X_2^2n} \]

and we find

\[ \frac{E^* D^*E^*E^*E^* - \sqrt{D^*E^*E^* - C^2D^*E^*}}{2D^*E^*} \leq \frac{E^*}{X_2^2n} \]

**3.2 Estimation of the Population Ratio of Mean for Current Occasion in the Presence of Non-Response on the First Occasion**

When there is non-response only on the first occasion, the minimum variance linear unbiased estimator for the population ratio on current occasion can be obtained as follows:

\[ \hat{R}_{21}^* = a(\hat{R}_{1u}^* - \hat{R}_{1m}^* ) + c\hat{R}_{2m}^* + (1 - c)\hat{R}_{2u}^* \]  \hspace{1cm} \text{where} \hspace{1cm} \hat{R}_{2m} = \frac{\bar{y}_{2m}}{\bar{x}_{2m}} \hspace{1cm} \text{and} \hspace{1cm} \hat{R}_{2u} = \frac{\bar{y}_{2u}}{\bar{x}_{2u}}

The variance of \( \hat{R}_{21}^* \) is given by

\[ V(\hat{R}_{21}^*) = a^2 \left( \frac{1}{q} + \frac{1}{p} \right) \frac{1}{nX_2^2} D^* + c^2 \frac{1}{mnX_2^2} B + (1 - c)^2 \frac{1}{qnX_2^2} B - 2ac \frac{1}{pnmX_2^2k_1} \]

which is minimum when

\[ a_{opt} = \frac{pq\bar{x}_1Bk_1}{X_2(D^*B - q^2k_1^2)} \hspace{1cm} \text{and} \hspace{1cm} c_{opt} = \frac{pD^*B}{D^*B - q^2k_1^2} \]

where
Thus the estimator \( \hat{R}_{21}^* \) turns out to be

\[
\hat{R}_{21}^* = \frac{pq \bar{x}_1 B k_1}{X_2 (D^* B - q^2 k_1^2)} (\bar{y}_{1u} - \bar{y}_{1m}) + \frac{pD^* B}{D^* B - q^2 k_1^2} \hat{R}_{2m} + (1 - \frac{pD^* B}{D^* B - q^2 k_1^2}) \hat{R}_{2u}
\]

with the variance

\[
V(\hat{R}_{21}^*) = \frac{B}{X_2^2 n} \frac{D^* B - q^2 k_1^2}{D^* B - q^2 k_1^2}
\]

while if \( W_2 = 0 \), i.e., there is non-response, the \( V(\hat{R}_{21}^*) \) reduces to

\[
V(\hat{R}_2) = \frac{B}{X_2^2 n} \frac{AB - q^2 k_1^2}{AB - q^2 k_1^2}
\]

where \( \hat{R}_2 \) is the usual estimator of the ratio of mean for the current occasion in the context of sampling on two occasions when there is complete response, that is,

\[
\hat{R}_2 = a \hat{R}_{1u} + b \hat{R}_{1m} + c \hat{R}_{2m} + d \hat{R}_{2u}
\]

The optimum fraction to be unmatched is given by

\[
q_{opt}^{(1)} = \frac{D^* B - \sqrt{D^* B - q^2 k_1^2 D^* B}}{k_1^2}
\]

and thus the minimum variance of \( \hat{R}_{21}^* \) is

\[
V_{\text{min}}(\hat{R}_{21}^*) = \frac{B}{X_2^2 n} \frac{D^* B + \sqrt{D^* B - q^2 k_1^2 D^* B}}{2 D^* B}
\]

### 3.3 Estimation of the Population Ratio of Mean for Current Occasion in the Presence of Non-Response on the Second Occasion

When there is non-response only on the second occasion, the minimum variance linear unbiased estimator for the population ratio on current occasion can be obtained as follows:

\[
\hat{R}_{22}^* = a (\hat{R}_{1u} - \hat{R}_{1m}) + c \hat{R}_{2m} + (1 - c) \hat{R}_{2u}
\]

where \( \hat{R}_{1m} = \bar{y}_{1m} / \bar{x}_{1m} \) and \( \hat{R}_{1u} = \bar{y}_{1u} / \bar{x}_{1u} \)

The variance of \( \hat{R}_{22}^* \) is given by

\[
V(\hat{R}_{22}^*) = a^2 \left( \frac{1}{q} + \frac{1}{p} \right) \frac{1}{nX_2^2} A + c^2 \frac{1}{pnX_2^2} E^* + (1 - c)^2 \frac{1}{qnX_2^2} E^* - 2ac \frac{1}{pnX_1X_2} k_1
\]

which is minimum when

\[
a_{opt} = \frac{pq \bar{x}_1 E^* k_1}{X_2 (AE^* - q^2 k_1^2)} \quad \text{and} \quad c_{opt} = \frac{pAE^*}{AE^* - q^2 k_1^2}
\]

where

\[
k_1 = \text{Cov} (y_1, y_2) - R_1 \text{Cov} (y_2, x_1) - R_2 \text{Cov} (y_1, x_2) + R_1 R_2 \text{Cov} (x_1, x_2)
\]
Thus the estimator $\hat{R}_{22}^*$ turns out to be

$$\hat{R}_{22}^* = \frac{pq \bar{X} l E^* k_1}{\bar{X}^2 (AE^*-q^2 k_1^2)} (\hat{R}_{1u} - \hat{R}_{1m}) + \frac{pAE^*}{AE^*-q^2 k_1^2} \hat{R}_{2m}^* + (1 - \frac{pAE^*}{AE^*-q^2 k_1^2}) \hat{R}_{2u}^*$$

with the variance

$$V(\hat{R}_{22}^*) = \frac{E^* E^* A^2 - q^2 E^* k_1^2}{\bar{X}^2 n (AE^*-q^2 k_1^2)}$$

while if $W = 0$, i.e., there is non-response, the $V(\hat{R}_{22}^*)$ reduces to

$$V(\hat{R}_2) = \frac{B \bar{X}^2 n (AB^*-q^2 k_1^2)}{\bar{X}^2 n (AB^*-q^2 k_1^2)}$$

where $\hat{R}_2$ is the usual estimator of the ratio of mean for the current occasion in the context of sampling on two occasions when there is complete response, that is,

$$\hat{R}_2 = a \hat{R}_{1u} + b \hat{R}_{1m} + c \hat{R}_{2m} + d \hat{R}_{2u}.$$  

The optimum fraction to be unmatched is given by

$$q_{opt}^{(2)} = \frac{AE^*-\sqrt{E^2 A^2 - k_1^2 E^* A}}{k_1^2}$$

and thus the minimum variance of $\hat{R}_{22}^*$ is

$$V_{min}(\hat{R}_{22}^*) = \frac{E^* E^* A^2 - q^2 E^* k_1^2}{\bar{X}^2 n (AE^*-q^2 k_1^2)}$$

3.4 Comparison Between Variances of the Estimators, $R_2, R_2^*, R_{21}^*$ and $R_{22}^*$

In this subsection, we carry out an analysis based on the loss in precision of $\hat{R}_2^*, \hat{R}_{21}^*$ and $\hat{R}_{22}^*$ with respect to $\hat{R}_2$. This loss is expressed in percentage and given by

$$L_{12} = \left[\frac{V(\hat{R}_2^*)}{V(\hat{R}_2)} - 1\right] \times 100, \quad L_1 = \left[\frac{V(\hat{R}_{21}^*)}{V(\hat{R}_2)} - 1\right] \times 100, \quad \text{and} \quad L_2 = \left[\frac{V(\hat{R}_{22}^*)}{V(\hat{R}_2)} - 1\right] \times 100,$$

respectively. Now, we assume that

$$C_{y_1} = C_{y_2} = C_{x_1} = C_{x_2} = C_0, \quad C_{y_1(2)} = C_{y_2(2)} = C_{x_1(2)} = C_{x_2(2)} = C_{0(2)}$$

$$\rho_1 = \rho_2 = \rho_3 = \rho_4 = \rho, \quad \rho_1(2) = \rho_2(2) = \rho_3(2) = \rho_4(2) = \rho(2)$$

$$\rho_5 = \rho_6 = \rho_0, \quad \rho_5(2) = \rho_6(2) = \rho_0(2)$$

The expressions of $D^*, E^*$ and $C^*$ becomes

$$D^* = 2\bar{Y}_1^2 d, \quad E^* = 2\bar{Y}_2^2 d, \quad \text{and} \quad C^* = 2\bar{Y}_1 \bar{Y}_2 t$$

where

$$d = (1 - \rho) C_0^2 + W_2 (k - 1) (1 - \rho(2)) C_{0(2)}^2, \quad t = (\rho_0 - \rho) C_0^2 + W_2 (k - 1) (\rho_0(2) - \rho(2)) C_{0(2)}^2$$

$$L_{12} = \left[\frac{d^2 - q t^2}{d^2 - q t^2} \frac{d}{d^2 - q t^2} \frac{(1 - \rho)^2 - (\rho_0 - \rho)^2 q^2}{(1 - \rho)^2 - (\rho_0 - \rho)^2 q^2} - 1\right] \times 100$$
The losses in precision of $R^{**}_2$, $R^{**}_{21}$ and $R^{**}_{22}$ with respect to $R_2$ for different values of $C_0$, $C_{0(2)}$, $\rho$, $\rho_0$, $\rho_{(2)}$ and $\rho_{0(2)}$, are presented in tables 1-2 and in Figure 1. It is assumed that $N = 300$ and $n = 50$. From these tables, we obtain the following conclusions:

1. For the case $C_0 < C_{0(2)}$, the loss in precision of $R^{**}_2$, $R^{**}_{21}$ and $R^{**}_{22}$ with respect to $R_2$ increases as the values of $C_{0(2)}$ increase; see Figure 1(a).

2. For the case $C_0 > C_{0(2)}$, the loss in precision of $R^{**}_2$, $R^{**}_{21}$ and $R^{**}_{22}$ with respect to $R_2$ decreases as the values of $C_0$ increase; see Figure 1(b).

3. For the case $C_0 = C_{0(2)}$, the loss in precision of all the estimators with respect to $R_2$ remain constant as the values of $C_0$ and $C_{0(2)}$ increase; see Figure 1(c).

4. For the case $\rho > \rho_0$, the loss in precision of all the estimators with respect to $R_2$ decreases as the values of $\rho_0$ increase; see Figure 1(d).

5. For the case $\rho < \rho_0$, the loss in precision of $R^{**}_{21}$ with respect to $R_2$ decreases as the values of $\rho$ increase, whereas the loss in precision of $R^{**}_2$ and $R^{**}_{22}$ with respect to $R_2$ increases as the values of $\rho$ increase; see Figure 1(e).

6. For the case $\rho = \rho_0$, the loss in precision of $R^{**}_2$ and $R^{**}_{22}$ with respect to $R_2$ increases as the values of $\rho$ and $\rho_0$ increase, whereas the loss in precision of $R^{**}_{21}$ remain constant as the values of $\rho$ and $\rho_0$ increase; see Figure 1(f).

7. For the case $\rho_{(2)} > \rho_{0(2)}$, the loss in precision of $R^{**}_2$ with respect to $R_2$ increases as the values of $\rho_{0(2)}$ increase, whereas the loss in precision of $R^{**}_{21}$ and $R^{**}_{22}$ with respect to $R_2$ remains constant as the values of $\rho_{0(2)}$ increase; see Figure 1(g).

8. For the case $\rho_{(2)} < \rho_{0(2)}$, the loss in precision $R^{**}_2$, $R^{**}_{21}$ and $R^{**}_{22}$ with respect to $R_2$ decreases as the values of $\rho_{0(2)}$ increase; see Figure 1(h).

9. For the case $\rho_{(2)} = \rho_{0(2)}$, the loss in precision of $R^{**}_2$, $R^{**}_{21}$ and $R^{**}_{22}$ with respect to $R_2$ decreases as the values of $\rho_{(2)}$ and $\rho_{0(2)}$ increase; see Figure 1(i).

10. The loss in precision of $R^{**}_2$, $R^{**}_{21}$ and $R^{**}_{22}$ with respect to $R_2$ increases as the values of $W_2$ increase; see Figure 1(j).

11. The loss in precision of $R^{**}_2$, $R^{**}_{21}$ and $R^{**}_{22}$ with respect to $R_2$ increases as the values of $(k - 1)$ increase; see Figure 1(k).

12. The loss in precision of $R^{**}_2$, $R^{**}_{21}$, $R^{**}_{22}$ with respect to $R_2$ increases as the values of $q$ increase; see Figure 1(l).
Fig. 1. Loss in precision, expressed in percentage of $\hat{R}_{22}^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$, with respect to $\hat{R}_2$ for (a)-(b) different values of $C_0$, (c) the case $C_0(2)=C_0$, (d)-(e) different values of $\rho_0$ and $\rho$, (f) the case $\rho=\rho_0$, (g)-(h) different values of $\rho(2)$ and $\rho_0(2)$, (i) the case $\rho(2)=\rho_0(2)$, (j)-(k) different values of $W_2$ and $(k-1)$ and (l) different values of $q$.

Table 1 Loss in precision, expressed in percentage, of $\hat{R}_{22}^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ for different values of $C_0$, $C_0(2)$, $\rho$, $\rho_0$, $\rho(2)$ and $\rho_0(2)$.

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Table 2 Loss in precision, expressed in percentage, of \(\hat{R}_{2}^{*\ast}, \hat{R}_{21}^{*\ast}\) and \(\hat{R}_{22}^{*\ast}\) with respect to \(\hat{R}_{2}\) for different values of \(W_2\), \((k-1)\) and \(q\).
4. Comparing Estimators in Terms of Survey Cost

We give some ideas about how saving in cost through mail surveys in the context of successive sampling on two occasions for different assumed values of $C_0$, $C_0(2)$, $\rho$, $\rho_0$, $\rho_0(2)$, $W_2$, $(k-1)$ and $q$. Let $N = 300$, $n = 50$, $c_0 = 1$, $c_1 = 4$, and $c_2 = 45$ (see Choudhary et al., p. 339) where $c_0$, $c_1$, and $c_2$ denote the cost per unit for mailing a questionnaire, processing the results from the first attempt respondents, and collecting data through personal interview, respectively. In addition, $C_{00}$ is the total cost incurred for collecting the data by personal interview from the whole sample, i.e., when there is no non-response. The cost function in this case is given by (assuming the cost incurred on data collection for the matched and unmatched portion of the sample are same and cost incurred on the data collection on both occasions is same)

$$C_{00} = 2nc_2.$$  \hspace{1cm} (6)

Substituting the values of $n$ and $c_2$ in Eq. (6), the total cost work out to be 4500.

Let $n_1$ denotes the number of units which respond at the first attempt and $n_2$ denotes the number of units which do not respond. Thus,

1. The cost function for the case when there is non-response on both occasions is

$$C_0^* = 2\left[c_0n + c_1n_1 + \frac{c_2n_2}{k-1}\right].$$

The expected cost is given by

$$E(C_0^*) = 2n_0^* \left[c_0 + c_1W_1 + \frac{c_2W_2}{k-1}\right],$$

where $W_1 = N_1/N$ and $W_2 = N_2/N$, such that $W_1 + W_2 = 1$ and

$$n_0^* = \left[\frac{d^2-aq^2}{d^2-q^2t^2} \frac{d}{(1-\rho)^2-(\rho_0-\rho)^2q^2}\right].$$

2. The cost function for the case when there is only non-response on the second occasion is

$$C_1^* = 2c_0n + c_1n + \left[c_1n_1 + \frac{c_2n_2}{k-1}\right]$$

and the expected cost is given by

$$E(C_1^*) = n_1^* \left[2c_0 + c_1(W_1 + 1) + \frac{c_2W_2}{k-1}\right].$$

where

$$n_1^* = \left[\frac{d(1-\rho)-2(\rho_0-\rho)^2q^2}{d(1-\rho)-2(\rho_0-\rho)^2C_0^2q^2} \frac{d}{(1-\rho)^2-(\rho_0-\rho)^2q^2}\right].$$

3. The cost function for the case when there is non-response on first occasion only is

$$C_2^* = \left[c_1n_1 + \frac{c_2n_2}{k-1}\right] + 2c_0n + c_1n,$$

which expected cost is expressed as

$$E(C_2^*) = n_2^* \left[2c_0 + c_1(W_1 + 1) + \frac{c_2W_2}{k-1}\right].$$

where

$$n_2^* = \left[\frac{d(1-\rho)-2(\rho_0-\rho)^2c_0^2q^2}{d(1-\rho)-2(\rho_0-\rho)^2C_0^2q^2} \frac{d}{(1-\rho)^2-(\rho_0-\rho)^2q^2}\right].$$
By equating the variances $\hat{R}_{2}^{* *}, \hat{R}_{21}^{* *}$, and $\hat{R}_{22}^{* *}$, respectively, to $V(\hat{R}_{2})$ and using the assumed values of different parameters, the values of the sample size for the three cases and the corresponding expected cost of survey were determined with respect of $\hat{R}_{2}^{* *}, \hat{R}_{21}^{* *}$, and $\hat{R}_{22}^{* *}$. The sample sizes associated with the three estimators which provide equal precision to the estimator $\hat{R}_{2}$ are denoted by $n_0^*, n_1^*$ and $n_2^*$. The results of this exercise are presented in tables 3-4 and in Figures 2-3. From these tables and figures, we obtain the following conclusions:

1. For the case $C_0 < C_0(2)$, the saving in cost for $\hat{R}_{2}^{* *}, \hat{R}_{21}^{* *}$ and $\hat{R}_{22}^{* *}$ decreases as the values of $C_0(2)$ increase; see Figure 2(a).
   The sample sizes for $\hat{R}_{2}^{* *}, \hat{R}_{21}^{* *}$ and $\hat{R}_{22}^{* *}$, which have the same precision than $\hat{R}_{2}$, increase as the values of $C_0(2)$ increase; see Figure 2(b).

2. For the case $C_0 > C_0(2)$, the saving in cost for $\hat{R}_{2}^{* *}, \hat{R}_{21}^{* *}$ and $\hat{R}_{22}^{* *}$ increases as the values of $C_0$ increase; see Figure 2(c).
   The sample sizes for $\hat{R}_{2}^{* *}, \hat{R}_{21}^{* *}$ and $\hat{R}_{22}^{* *}$ which have the same precision than $\hat{R}_{2}$, decrease as the values of $C_0$ increase; see Figure 2(d).

3. For the case $C_0 = C_0(2)$ the saving in cost for all the estimators remains constant as the values of $C_0$ and $C_0(2)$ increase; see Figure 2(e).
   The sample sizes for all the estimators, which have the same precision than $\hat{R}_{2}$, remain constant as the values of $C_0$ and $C_0(2)$ increase; see Figure 2(f).

4. For the case $\rho > \rho_0$, the saving in cost for all the estimators increases as the values of $\rho_0$ increase; see Figure 2(g).
   The sample sizes for the three estimators, which have the same precision than $\hat{R}_{2}$, decreases as the values of $\rho_0$ increase; see Figure 2(h).

5. For the case $\rho < \rho_0$, the saving in cost for $\hat{R}_{2}^{* *}$ and $\hat{R}_{22}^{* *}$ the saving in cost decreases as the values of $\rho$ increase, whereas for $\hat{R}_{21}^{* *}$ the saving in cost increases as the values of $\rho$ increase; see Figure 2(i).
   The sample sizes for $\hat{R}_{2}^{* *}$ and $\hat{R}_{22}^{* *}$, which give equal precision to $\hat{R}_{2}$ increase as the values of $\rho$ increase, whereas the sample size for $\hat{R}_{21}^{* *}$, which has the same precision than $\hat{R}_{2}$, remains constant as the values of $\rho$ increase; see Figure 2(j).

6. For the case $\rho = \rho_0$, the saving in cost for all the estimators decreases as the values of $\rho$ and $\rho_0$ increase; see Figure 2(k).
   The sample sizes for the three estimators, which have the same precision than $\hat{R}_{2}$, increases as the values of $\rho$ and $\rho_0$ increase; see Figure 2(l).

7. For the case $\rho(2) > \rho_0(2)$, the saving in cost for $\hat{R}_{2}^{* *}$ decreases as the values of $\rho_0(2)$ increase, whereas for $\hat{R}_{21}^{* *}$ and $\hat{R}_{22}^{* *}$ the saving in cost remains constant as the values of $\rho_0(2)$ increase; see Figure 3(a).
   The sample size for $\hat{R}_{2}^{* *}$, which have the same precision than $\hat{R}_{2}$, increases as the values of $\rho_0(2)$ increase, whereas for $\hat{R}_{21}^{* *}$ and $\hat{R}_{22}^{* *}$ which give equal precision to $\hat{R}_{2}$ remains constant as the values of $\rho_0(2)$ increase; see Figure 3(b).
8. For the case $\rho(2) < \rho_0(2)$, the saving in cost for all the estimators increases as the values of $\rho(2)$ increase; see Figure 3(c).

The sample sizes for $\hat{R}_2^{**}$ and $\hat{R}_{22}^{**}$, which have the same precision than $\hat{R}_2$ decreases as the values of $\rho(2)$ increase, whereas the sample size for $\hat{R}_{21}^{**}$, which have the same precision than $\hat{R}_2$ remains constant as the values of $\rho(2)$ increase; see Figure 3(d).

9. For the case $\rho(2) = \rho_0(2)$, the saving in cost for all the estimators increases as the values of $\rho(2)$ and $\rho_0(2)$ increase; see Figure 3(e).

The sample sizes for $\hat{R}_2^{**}, \hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$, which have the same precision than $\hat{R}_2$, decrease as the values of $\rho(2)$ and $\rho_0(2)$ increase; see Figure 3(f).

10. The saving in cost for $\hat{R}_2^{**}, \hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ decreases as the values of $W_2$ increase; see Figure 3(g).

The sample sizes for $\hat{R}_2^{**}, \hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$, which have the same precision than $\hat{R}_2$, increases as the values of $W_2$ increase; see Figure 3(h).

11. The saving in cost for $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ increases as the values of $(k - 1)$ increase whereas for $\hat{R}_{22}^{**}$ the saving in cost decreases as the values of $k - 1$ increase; see Figure 3(i).

The sample sizes for $\hat{R}_2^{**}, \hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$, which have the same precision than $\hat{R}_2$, increases as the values of $k - 1$ increase; see Figure 3(j).

12. The saving in cost for all the estimators decreases as the values of $q$ increase; see Figure 3(k).

The sample sizes for $\hat{R}_2^{**}, \hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$, which have the same precision than $\hat{R}_2$, increases as the values of $q$ increase; see Figure 3(l).
Fig. 2. Corresponding expected cost of survey and sample sizes, which have the same precision than $R_2^{**}$, $R_{21}^{**}$ and $R_{22}^{**}$, with respect to $R_2$ for (a)-(b) different values of $C_{0(2)}$, (c)-(d) different values of $C_0$, (e)-(f) the case $C_{0(2)}=C_0$, (g)-(h) different values of $\rho_0$, (i)-(j) different values of $\rho$ and (k)-(l) the case $\rho=\rho_0$. 
Fig. 3. Corresponding expected cost of survey and sample sizes, which have the same precision than \( \hat{R}_{2}^{*}, \hat{R}_{21}^{*}, \) and \( \hat{R}_{22}^{*}, \) with respect to \( \hat{R}_{2} \) for (a)-(b) different values of \( \rho(2), \) (c)-(d) different values of \( \rho(2), \) (e)-(f) the case \( \rho(2) = \rho(2), \) (g)-(h) different values of \( W_{2} \), (i)-(j) different values of \( (k - 1) \) and (k)-(l) different values of \( q. \)

Table 3 Sample sizes and corresponding expected cost of survey, which have the same precision than \( \hat{R}_{2}^{*}, \hat{R}_{21}^{*}, \) and \( \hat{R}_{22}^{*}, \) with respect to \( \hat{R}_{2} \) for different values of \( C_{0}, C_{0(2)}, \rho, \rho_{0}, \rho_{(2)} \) and \( \rho_{0(2)}. \)

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### Table 4

Sample sizes and corresponding expected cost of survey, which have the same precision than $\hat{R}_{2}^{*}$, $\hat{R}_{21}$ and $\hat{R}_{22}^{*}$, with respect to $\hat{R}_{2}$ for different values of $W_2$, $(k - 1)$ and $q$.

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### Note

$\rho > \rho_0$, $\rho < \rho_0$, $\rho = \rho_0$, $\rho_{(2)} > \rho_{0(2)}$, $\rho_{(2)} < \rho_{0(2)}$, $\rho_{(2)} = \rho_{0(2)}$, with respect to $\hat{R}_{2}$ for different values of $W_2$, $(k - 1)$ and $q$. 
5. Conclusions

In this paper, we have used the HH technique for estimating the population ratio of mean in mail surveys. This problem is conducted for current occasion in the context of sampling on two occasions when there is non-response (i) on both occasions, (ii) only on the first occasion and (iii) only on the second occasion. The results obtained reveals that the loss in precision is maximum for the estimation of the ratio of mean when there is non-response only on the second occasion, whereas it is least for the estimation of the ratio of mean when there is non-response on both occasions and when there is non-response only on the first occasion. Also, we derive the sample sizes and the saving in cost for all the estimators, which have the same precision than the estimator of the population ratio of mean when there is no non-response. In the majority of the cases the sample sizes and the saving in cost is maximum for the estimation of the ratio of mean when there is non-response on both occasions, whereas it is least for the estimation of the ratio of mean when there is non-response only on the first occasion and when there is non-response only on the second occasion.

References


