Use of Some Known Values of Population Parameters for Estimating the Finite Population Mean for Random Non-response in Survey Sampling

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Received 26 May 2014; Published online 30 May 2015

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Abstract

In the present paper a family of estimators for estimating population mean by using known values of some population parameters viz. standard deviation ($S_x$), coefficient of variation ($C_x$), skewness ($\beta_1(x)$), kurtosis ($\beta_2(x)$), correlation coefficient ($\rho$) of the population is studied under an assumption that the number of sampling units on which information cannot be obtained due to random non response follows some distribution. The properties of suggested estimators are studied and their comparison is also studied.

Keywords: Bias, mean square error, study variable, auxiliary variable, random non response.

1. Introduction

The use of auxiliary information in survey sampling has its own eminent role. The ratio estimator, product estimator and regression estimator are well known examples. To obtain the most efficient estimator, many authors proposed ratio and product estimators using the standard deviation, coefficient of variation, skewness, kurtosis, correlation coefficient, etc. of the auxiliary variable.

Consider a finite population $U = (U_1, U_2, ..., U_N)$ of $N$ identifiable units taking values $(y_1, y_2, ..., y_N)$ on a study variable. The use of auxiliary variable in survey sampling has its own eminent role. The ratio, product and regression estimator are well known examples. Following Olkin (1958), Isaki (1983) has considered the use of auxiliary variable in building up ratio and regression estimators for estimating the population variance.

Let $\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i$ and $S_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X})^2$ be the known population mean and variance of the
auxiliary variable $x$. Assuming that a simple random sample of size $n$ is drawn from $U$. Defining

$$s^2_y = \frac{1}{n-1} \sum_{i=1}^{n}(y_i - \bar{y})^2, \; \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i, \; s_x^2 = \frac{1}{n-1} \sum_{i=1}^{n}(x_i - \bar{x})^2, \; \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i.$$ 

Tracy and Osahan (1994) and Singh, Joarder and Tracy (2000) studied the effect of random non response:

(i) On the study as well as the auxiliary variable (Situation I), and

(ii) On the study variable only (Situation II), on the usual ratio and regression estimators of the population mean.

Singh and Joarder (1998) studied the effect of random non response on the study and auxiliary variables on several estimators of variance and also the distribution for the number of sampling units on which random non response is present.

2. Distribution of Random Non-response and Some Expected Values

Let $U$: $(U_1, U_2, ..., U_N)$ denote the population of $N$ units from which a simple random sample of size $n$ is drawn without replacement. If $r$ $= 0, 1, 2, ..., (n - 2)$ denotes the number of sampling units on which information could not be obtained due to random non response, then the remaining $(n - r)$ units in the sample can be treated as SRSWOR sample from $U$. Assume that $r$ should be less than $(n - 1)$ and if $p$ denotes the probability of non response among the $(n - 2)$ possible values of non-response, then $r$ has the following discrete distribution given by

$$P(r) = \frac{n-r}{nq + 2p} \binom{n-2}{r} p^r q^{n-2-r}, \quad (2.1)$$

where $q = 1 - p$ and $r = 0, 1, 2, ..., (n - 2)$. Let us define $\bar{y}_{n-r} = \bar{Y}(1 - \epsilon) ; \; \bar{x}_{n-r} = \bar{X}(1 - \delta)$ ;

$$\bar{y}_{n-r} = \frac{1}{n-r} \sum_{i=1}^{n-r} y_i \; \text{and} \; \bar{x}_{n-r} = \frac{1}{n-r} \sum_{i=1}^{n-r} x_i$$

have their usual meaning (Tracy and Osahan (1994)). Thus, under the probability model given by (2.1), we have the following results:

$$E(\epsilon) = E(\delta) = 0; \quad E(\epsilon^2) = \left(\frac{1}{nq+2p} - \frac{1}{N}\right) C_y^2, \; E(\delta^2) = \left(\frac{1}{nq+2p} - \frac{1}{N}\right) C_x^2, \; E(\epsilon\delta) = \left(\frac{1}{nq+2p} - \frac{1}{N}\right) \rho C_y C_x;$$

where $C_y = S_y / \bar{Y}$ ; $C_x = S_x / \bar{X}$ and $\rho = S_{yx}/S_x S_y$ have their usual meaning. It is interesting to note that under model (2.1), the above expected values are exact and hence makes valid comparison with the estimators in the absence of non-response.

In the present study I have suggested a class of estimators for the population mean in the situation when there is random non response on study and auxiliary variables.
3. The Suggested Family of Estimators

Following Khoshnevisan et al. (2007) and Koyunco and Kadilar (2009), a family of estimators for the population mean in simple random sampling without replacement (SRSWOR) is proposed when random non response exists on both the study variable \( y \) and the auxiliary variable \( x \) and population mean \( \bar{X} \) of the auxiliary variable is known, is given by

\[
t^* = \bar{y}_{n-r} \left\{ \frac{a\bar{X} + b}{(ax_{n-r} + b) + (1-ax_{n+b})} \right\}^g,
\]

(3.1)

where \( a \neq 0, b \) are either real number or functions of the known parameters of the auxiliary variable \( x \) such as the standard deviation \( (S_x) \), coefficient of variation \( (C_x) \), Skewness \( (\beta_1(x)) \) and kurtosis \( (\beta_2(x)) \) and the correlation coefficient \( (\rho) \) of the population. Here \( g \) and \( \alpha \) are suitably chosen scalars such that the mean square error of \( t^* \) is minimum.

**Table 1** Some members of the family of estimators of \( t^* \).

<table>
<thead>
<tr>
<th>Ratio Estimators ((g = 1))</th>
<th>Product Estimators ((g = -1))</th>
<th>(\alpha)</th>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_1^* = \bar{y}<em>{n-r} \left( \frac{\bar{X}}{\bar{x}</em>{n-r}} \right))</td>
<td>(t_2^* = \bar{y}<em>{n-r} \left( \frac{\bar{x}</em>{n-r}}{\bar{X}} \right))</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(t_3^* = \bar{y}<em>{n-r} \left( \frac{\bar{X} + C_x}{\bar{x}</em>{n-r} + C_x} \right))</td>
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<td>(t_6^* = \bar{y}<em>{n-r} \left( \frac{\beta_2(x)\bar{x}</em>{n-r} + C_x}{\beta_2(x)\bar{X} + C_x} \right))</td>
<td>1</td>
<td>(\beta_2(x))</td>
<td>(C_x)</td>
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<tr>
<td>(t_7^* = \bar{y}<em>{n-r} \left( \frac{C_x\bar{X} + \beta_2(x)}{C_x\bar{x}</em>{n-r} + \beta_2(x)} \right))</td>
<td>(t_8^* = \bar{y}<em>{n-r} \left( \frac{C_x\bar{x}</em>{n-r} + \beta_2(x)}{C_x\bar{X} + \beta_2(x)} \right))</td>
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<td>1</td>
<td>1</td>
<td>(\beta_2(x))</td>
</tr>
</tbody>
</table>

Thus, the bias and MSE of \( t^* \) to the first degree of approximation is given by

\[
B(t^*) = \left( \frac{1}{nq+2p} - \frac{1}{N} \right) \bar{y} \left\{ \frac{g(g+1)}{2} \alpha^2 \nu^2 C_x^2 - \alpha g C_{yx} \right\},
\]

(3.2)

\[
MSE(t^*) = \left( \frac{1}{nq+2p} - \frac{1}{N} \right) \left\{ S_x^2 + \alpha^2 \nu^2 g^2 S_x^2 R^2 - 2\alpha g RS_{yx} \right\},
\]

(3.3)
where $\nu = \frac{ax}{ax + b}$

which is minimum when $\alpha = \frac{K}{\nu g}$; $K = \frac{S_{yx}}{S_{x}^2}$.

Thus the minimum MSE of $t^*$ is given by

$$
\text{min. MSE}(t^*) = \left(\frac{1}{nq+2p} - \frac{1}{N}\right) (1 - \rho^2) S_y^2,
$$

which is equal to the MSE of regression type estimator, i.e. $t_{tr} = \bar{y}_{n-r} + a(\bar{x} - \bar{x}_{n-r})$ given by Singh et al. (2000).

The ratio estimators, which are given in Table 1, are in the same family (3.1) and the mean square error (3.3) for these estimators is as follows:

$$
\text{MSE}(t_i^*) = \left\{ \begin{array}{ll}
\left(\frac{1}{nq+2p} - \frac{1}{N}\right) \{S_y^2 + R^2 S_x^2 - 2RS_{yx}\}; & i = 1 \\
\left(\frac{1}{nq+2p} - \frac{1}{N}\right) \{S_y^2 + \nu \frac{2}{(i-1)} R^2 S_x^2 - 2\nu \frac{(i-1)}{i} RS_{yx}\}; & i = 3, 5, 7, ... 17.
\end{array} \right.
$$

For product estimators, the MSE is given by

$$
\text{MSE}(t_j^*) = \left\{ \begin{array}{ll}
\left(\frac{1}{nq+2p} - \frac{1}{N}\right) \{S_y^2 + R^2 S_x^2 + 2RS_{yx}\}; & j = 2 \\
\left(\frac{1}{nq+2p} - \frac{1}{N}\right) \{S_y^2 + \nu \frac{2}{(i-1)} R^2 S_x^2 + 2\nu \frac{(i-1)}{i} RS_{yx}\}; & j = 4, 6, 8, ... 18.
\end{array} \right.
$$

Now, motivated by Prasad (1989) and Gandge et al. (1993), a new family of estimators is proposed in the presence of random non response on study as well as auxiliary variables and population mean of auxiliary variable is known, as

$$
\eta^* = k\bar{y}_{n-r} \left( \frac{ax+b}{\alpha(ax_{n-r}+b)+(1-\alpha)(ax+b)} \right)^g
$$

(3.7)

The bias and MSE of the proposed estimator $\eta^*$ is given by

$$
\text{B(}\eta^*) = \left(\frac{1}{nq+2p} - \frac{1}{N}\right) \left[ k\bar{y} \left( \frac{g(g+1)}{2} - \alpha^2 \nu g C_x^2 - g\alpha \nu C_{yx} \right) \right] + \bar{y}(k - 1)
$$

(3.8)

$$
\text{MSE}(\eta^*) = \left(\frac{1}{nq+2p} - \frac{1}{N}\right) \left[ k^2 S_y^2 + \{k^2(2g^2 + g) - k(g^2 + g)\} \alpha^2 \nu^2 R^2 S_x^2 - 2g\alpha \nu (2k^2 - k) RS_{yx} \right] + \bar{y}^2 (k - 1)^2
$$

(3.9)

which is minimum, when
For the ratio estimators as given in Table 2, we can express the MSE given in (3.9) by the following equations

\[ k = \frac{A}{2B} = k^* \text{(say)} \]  

where

\[ A = \left( \frac{1}{nq + 2p} - \frac{1}{N} \right) \left\{ (g^2 + g)\alpha^2 \nu^2 R^2 S_x^2 - 2g\alpha vR S_{yx} \right\} + 2 \]

\[ B = \left( \frac{1}{nq+2p} - \frac{1}{N} \right) \left( S_x^2 + (2g^2 + g)\alpha^2 \nu^2 R^2 S_x^2 + 4g\alpha vR S_{yx} \right) + 1. \]

Thus, the minimum MSE of \( \eta^* \) is obtained as

\[ \text{min. MSE} (\eta^*) = \bar{Y}^2 \left( 1 - \frac{A^2}{4B} \right). \]  

Table 2 Some members of the family of estimators of \( \eta^* \)

<table>
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<tr>
<th>Ratio Estimators ( (g = 1) )</th>
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\[
MSE(\eta_i^*) = \begin{cases} 
\left(\frac{1}{nq+2p} - \frac{1}{N}\right) \left\{ k^2 S_y^2 + (3k^2 - 2k^*) R^2 S_x^2 - 2(2k^2 - k^*) R S_x \right\} + \bar{Y}^2 (k^* - 1)^2; i = 1 \\
\left(\frac{1}{nq+2p} - \frac{1}{N}\right) \left\{ k^2 S_y^2 + (3k^2 - 2k^*) \frac{v^2}{(i-1)^2} R^2 S_x^2 - 2v \frac{(i-1)^2}{(i-1)^2} (2k^2 - k^*) R S_x \right\} + \bar{Y}^2 (k^* - 1)^2; i = 3, 5, ..., 17 
\end{cases}
\]

which is minimum, when

\[
k^* = \frac{A^*}{B^*}; \quad A^* = \left(\frac{1}{nq+2p} - \frac{1}{N}\right) (R^2 S_x^2 - R S_x) + 1; \\
B^* = \left(\frac{1}{nq+2p} - \frac{1}{N}\right) (S_y^2 + 3R^2 S_x^2 - 4R S_x) + 1; \\
k^+ = \frac{A^+}{B^+}; \quad A^+ = \left(\frac{1}{nq+2p} - \frac{1}{N}\right) \left( \frac{v^2}{(i-1)^2} R^2 S_x^2 - v \frac{(i-1)^2}{(i-1)^2} R S_x \right) + 1; \\
B^+ = \left(\frac{1}{nq+2p} - \frac{1}{N}\right) (S_y^2 + 3\frac{v^2}{(i-1)^2} R^2 S_x^2 - 4v \frac{(i-1)^2}{(i-1)^2} R S_x) + 1.
\]

For the product estimators, the MSE’s are given as

\[
MSE(\eta_j^*) = \begin{cases} 
\left(\frac{1}{nq+2p} - \frac{1}{N}\right) \left\{ k^2 S_y^2 + k^0 R^2 S_x^2 - 2(2k^0 - k^0) R^2 S_y^2 \right\} + \bar{Y}^2 (k^0 - 1)^2; j = 2 \\
\left(\frac{1}{nq+2p} - \frac{1}{N}\right) \left\{ k^2 S_y^2 + k^2 \frac{v^2}{(i-1)^2} R^2 S_x^2 - 2v \frac{(i-1)^2}{(i-1)^2} (2k^2 - k^0) R^2 S_y \right\} + \bar{Y}^2 (k^0 - 1)^2; j = 4, 6, ..., 18 
\end{cases}
\]

Substituting the optimal values of \(k^*\) and \(k^+\) in (3.12), we get the minimum MSE of \(\eta_i^*\) as

\[
\text{min. } MSE(\eta_i^*) = \begin{cases} 
\left(1 - \frac{A^2}{B^*}\right); i = 1 \\
\left(1 - \frac{A^2}{B^*}\right); i = 3, 5, ..., 17 
\end{cases}
\]
Minimize the MSE of \( \eta_j^* \) with respect to \( k^0 \) and \( k' \), we get

\[
k^0 = \frac{A^0}{B^0}; \quad A^0 = \left( \frac{1}{nq + 2p} - \frac{1}{N} \right) RS_{yx};
\]

\[
B^* = \left( \frac{1}{nq + 2p} - \frac{1}{N} \right) \left( S_y^2 + R^2 S_x^2 + 4RS_{yx} \right) + 1;
\]

\[
k' = \frac{A'}{B'}; \quad A' = \left( \frac{1}{nq + 2p} - \frac{1}{N} \right) R\nu(i-1)S_{yx} + 1;
\]

\[
B' = \left( \frac{1}{nq + 2p} - \frac{1}{N} \right) \left( S_y^2 + \nu(i-1)R^2 S_x^2 + 4\nu(i-1)RS_{yx} \right) + 1.
\]

Substituting the optimal values of \( k^0 \) and \( k' \) in (3.13), we get the minimum MSE of \( \eta_j^* \) as

\[
\min. \text{MSE}(\eta_j^*) = \begin{cases} 
(1 - \frac{A^0}{B^0}); & j = 2 \\
(1 - \frac{A^2}{B'^2}); & i = 4, 6, ..., 18 
\end{cases} \tag{3.15}
\]

4. Efficiency Comparison

The \( t^* \)-family of estimators is more efficient than the classical ratio estimator, if

\[
\text{MSE}(t_i^*) < \text{MSE}(t_j^*); \quad i = 3, 5, 7, ..., 17,
\]

if

\[
\nu(i-1) < \frac{2S_{yx}}{RS_x} - 1 \text{ for } \nu(i-1) > 1
\]

\[
\nu(i-1) > \frac{2S_{yx}}{RS_x} - 1 \text{ for } \nu(i-1) < 1
\] \tag{4.1}

When (4.1) holds true, then the \( t^* \)-family of estimator is more efficient than the classical ratio estimator.

From (3.14) and (3.5) for \( i = 1 \) i.e. \( \text{MSE}(t_1^*) \), we have

\[
\min. \text{MSE}(\eta_i^*) < \text{MSE}(t_1^*); \quad i = 3, 5, 7, ..., 17,
\]

if

\[
\left( 1 - \frac{A^2}{B^*} \right) < \left( \frac{1}{nq + 2p} - \frac{1}{N} \right) \left( S_y^2 + R^2 S_x^2 - 2RS_{yx} \right).
\] \tag{4.2}

When (4.2) is satisfied, the suggested family is more efficient than the classical ratio estimator.

From (3.14), the suggested family of estimator is more efficient than the Prasad’s ratio estimator, if
\[
\text{min. MSE}(\eta_i^* < \text{min. MSE}(\eta_i^*); i = 3, 5, 7, \ldots, 17, \quad (4.3)
\]

When (4.3) is satisfied, the suggested family of estimators is more efficient than the Prasad’s ratio estimator.

From (3.14) and (3.5), we obtain

\[
\text{min. MSE}(\eta_1^*) < \text{MSE}(t_1^*)
\]

if

\[
\left(1 - \frac{A^2}{B^*}\right) < \left(\frac{1}{nq + 2p} - \frac{1}{N}\right)\left(S_y^2 + R^2S_x^2 - 2RS_{yx}\right)
\]

and

\[
\text{min. MSE}(\eta_i^*) < \text{MSE}(t_i^*); i = 3, 5, 7, \ldots, 17,
\]

if

\[
\left(1 - \frac{A^2}{B^*}\right) < \left(\frac{1}{nq + 2p} - \frac{1}{N}\right)\left(S_y^2 + R^2S_x^2 - 2R\left(\frac{j-1}{\sqrt{1}}\right)S_{yx}\right).
\]

From (3.4) and (3.11), we get

\[
\text{min. MSE}(\eta^*) < \text{MSE}(t^*)
\]

if

\[
\left(1 - \frac{A^2}{4B}\right) < \left(\frac{1}{nq + 2p} - \frac{1}{N}\right)\left(1 - \rho^2\right)S_y^2.
\]

5. Conclusion

A general family of estimators are proposed for estimating the population mean by using known values of some population parameters when the number of sampling units on which information cannot be obtained due to random non response on study as well as on the auxiliary variables. Motivated by Prasad (1989) and Gandge et al. (1993), a new family of estimator is proposed and theoretical shown that the proposed family of estimators have less mean square error as compared to the family of estimators based on Khoshnevisan et al. (2007) and Koyuncu and Kadilar (2009) estimators in the presence of random non response. The proposed family of estimator is more efficient than the usual ratio type estimator and the optimum MSE of the proposed estimator \(t^*\) is same to the MSE of Singh et al. (2000) regression type estimator.

Acknowledgement

Author is thankful to the learned referees for their valuable comments in improving the paper to its present form.
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