Improvements in the Blast-Mitigation Performance of Light-Tactical-Vehicle Side-Vent-Channel Solution using Aluminum-Foam Core Sandwich Structures

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Abstract

In the present work, an attempt is made to improve the blast-mitigation efficiency of our recently proposed light-tactical-vehicle side-vent-channel concept/solution. This concept/solution involves the use of side-vent-channels attached to the V-shaped vehicle underbody, and was motivated by the physical concepts and working principles of operation of the so-called “pulse detonation” rocket engines. By proper shaping of the V-hull and side-vent-channels, venting of supersonically-expanding gaseous detonation products is promoted in order to generate a downward thrust on the targeted vehicle. This, in turn, helps mitigate the blast-loads resulting from a shallow-buried mine detonated underneath a light tactical vehicle. The utility and the blast-mitigation capacity of this concept were examined in our prior work using several computational and design-optimization methods and tools. The results obtained show that the side-vent-channel solution has a limited (but noteworthy) blast-mitigation capacity. In the present work, an attempt is made to improve the blast-mitigation capacity of the side-vent-channel solution by substituting side-vent-channels made of sandwich-structures (consisting of steel face-sheets and aluminum-foam core) for all-steel side-vent-channels. The results obtained revealed that the use of sandwich-structures can improve blast-mitigation performance of the side-vent-channel solution, and that this improvement can be further increased through proper grading of the aluminum-foam density profile through the sandwich-structure core.

Keywords: Side-Vent-Channel Blast-Mitigation Concept/Solution, Aluminum-Foam Core Sandwich-Structures

1. Introduction

In the present work, the recently proposed side-vent-channel-based solution (Grujicic et al. 2013b,c) for improved survivability of light tactical military vehicles to beneath-underbody shallow-buried mine detonation has been upgraded. The main aspect of this upgrade involves replacement of the all-
steel side-vent channels with their sandwich-structure counterparts (structures consisting of steel face-sheets and graded aluminum-foam core). In addition, an engineering-optimization analysis is employed to identify an optimum foam-density grading profile through the sandwich-structure core. Two topics [(a) identification of the key limitations of the light tactical vehicles employed in recent and ongoing conflicts; and (b) the interaction of the soil ejecta and gaseous detonation products, resulting from the detonation of shallow-buried mines, with the target structure, and elucidation/quantification of the resulting impulse loading] which are closely related to the present subject matter have been reviewed in our recent work (Grujicic et al. 2009a,b) and hence, will not be covered here. Rather, in the remainder of this section, brief overviews will be provided of two other topics closely related to the present work: (a) the side-vent-channel-based blast-mitigation concept; and (b) our prior computational work/analysis aimed at establishing and maximizing the blast-mitigation potential of the side-vent-channel solution.

1.1. V-hull and Side-Vent-Channel Blast-Mitigation Concepts

To address the limitations of light tactical vehicles currently in use, the US military continues to seek innovative concepts and solutions which can: (a) improve blast-survivability of these vehicles; and (b) do so without compromising vehicle mobility/maneuverability, transportability, deployability or fuel economy. One of the concepts currently being used in the light tactical vehicles is the V-shaped vehicle hull (or simply V-hull). The two most common renditions of the V-hull are depicted schematically in Figures 1(a)–(b). In the case of the standard V-hull solution, Figure 1(a), the blast-mitigation performance (as measured by the reduction in the momentum transferred to the vehicle by the gaseous detonation products, soil ejecta and mine casing) is improved as the V-hull is made steeper. However, the maximum steepness is constrained by the requirements related to the minimum acceptable vehicle ground-clearance and the maximum acceptable cabin-floor height. In the case of the truncated V-hull design, Figure 1(b), V-hull steepness is increased at the expense of introducing a flat bottom portion of the V-hull. Due to the trade-off between the benefits (i.e. decreased blast impulse) offered by the increased steepness of the V-hull and the penalty (i.e. increased blast impulse) incurred due to the small flat section, and depending on the location of the detonated mine or IED, this design may or may not result in an improved blast-mitigation performance relative to that offered by the standard V-hull design (both associated with the same vehicle ground-clearance and cabin-floor height).

![Fig. 1. Two most common renditions of the V-shaped hull concept/solution: (a) standard V-shaped hull; and (b) truncated V-shaped hull](image-url)
The V-hull concept was further advanced in our recent work (Grujicic et al. 2013b,c) by attaching to it a series of side-vent-channels. The resulting concept has been termed the side-vent-channel solution. As shown schematically in Figure 2, the side-vent-channel solution utilizes flared tubular side-vent-channels (of the appropriate cross-sectional shape and wall-thickness) open at both ends. (It should be noted that in order to prevent potential misuse of the ideas proposed and the results obtained in the present work, the term “vehicle” has been replaced in Figure 2 as well as in the remainder of the manuscript with the term “surrogate box structure,” SBS.) The bottom end of each tube is cut parallel to the ground (to promote inflow of the detonation by-products and soil ejecta, and to prevent crushing/crumpling of the tube inlet under blast loads) and flush with the V-hull bottom. The channels/tubes are intended to function as exhaust nozzles as in the case of the pulse-detonation engine and, thus, provide a downward thrust to the SBS (through the gaseous-detonation-products supersonic-expansion effects). The additional role of the side-vent-channels is to reduce the blast momentum transferred to the targeted SBS by improving the venting of the gaseous detonation products, soil ejecta and mine-casing fragments (and, thus, by enhancing the downward-thrust effects).

Fig. 2. Side channels/tubes based blast-mitigation concept originally proposed by Grujicic et al. (2013b). Note that the abbreviation SBS stands for “surrogate box structure.”

1.2. Computational Verification and Validation of the Side-Vent-Channel Concept

To verify/validate the side-vent-channel concept, detailed computational analyses involving detonation of a landmine (of a prototypical shape, size and depth of burial, DOB) buried underneath the vent-channel-equipped SBS, and the interaction of the air-blast, soil ejecta, and mine-casing fragments with the SBS have been conducted by Grujicic et al. (2013b,c). A brief description of the computational analyses carried out in each of the Refs. (Grujicic et al. 2013b,c) is presented below. It should be noted that the computational analyses presented in Grujicic et al. (2013b,c) reveal not only the advancement in the side-vent-channel blast-mitigation concept, but also in the fidelity/physical-reality of the computational methods and tools used.
The computational analysis employed by Grujicic et al. (2013b) was of a combined Lagrangian/Eulerian fluid-structure interaction (FSI) type, within which both the gaseous materials (i.e. air and detonation products) and non-SBS solid materials (i.e. soil and mine fragments) are modeled as a multi-component Eulerian material. Consequently, this analysis suffered from at least the following two major deficiencies: (a) inaccuracies/uncertainties related to the definition of the boundaries/interfaces between different components of the Eulerian material, and between the Eulerian and SBS-Lagrangian materials; and (b) inability to take into account the granular/discrete character of the soil ejecta. Despite these deficiencies, the computational analyses carried out by Grujicic et al. (2013b) established, at least in semi-quantitative terms, that the side-vent-channels: (i) lower the blast impulse transferred to the vehicle, by promoting venting of soil ejecta, gaseous detonation-products and mine-casing fragments resulting from a mine-blast underneath the vehicle; (ii) lower the possibility for the vehicle lift-off from the ground, by promoting supersonic expansion of gaseous detonation-products exiting the channel, and thereby helping to create a downward thrust on the vehicle; (iii) do not limit the mobility of the occupants within the vehicle and their ability to survey the surroundings, since the side-vent-channels do not pass through the SBS cabin; and (iv) do not compromise SBS off-road structural reliability and durability, since the side-vent-channels attached to the V-hull do not considerably increase SBS-frame rigidity and, thus, do not significantly increase the rate of vehicle-frame fatigue-induced failure.

The aforementioned deficiencies of the combined Eulerian/Lagrangian analysis employed in the computational investigation of the side-vent-channel concept in Grujicic et al. (2013b) were addressed in Grujicic et al. (2013c). In Grujicic et al. (2013c), the side-vent-channel concept/solution was re-examined using combined finite-element/discrete-particle computational methods and tools. Within this approach, all non-SBS materials are treated as assemblies of discrete particles. While this approach helped remove some of the aforementioned deficiencies of the combined Eulerian/Lagrangian formulation related to the soil granularity, it introduced additional problems related to: (a) the treatment of discrete materials containing vastly different sizes of the constituent particles (i.e. molecules in air and in the gaseous detonation products, and grains/fragments in the case of soil and mine-casing); and (b) the ability to model hypersonic-expansion effects accompanying venting of the gaseous detonation products, as well as the generation of downward thrust on the SBS. Despite these deficiencies, the combined finite-element/discrete-particle analysis reconfirmed the aforementioned findings (yielded by the combined Eulerian/Lagrangian computational analysis (Grujicic et al. 2013b)) regarding the blast-mitigation utility and efficacy of the side-vent-channel solution. In addition, a parametric study for the geometry of the side-vent-channels was conducted in Ref. (Grujicic et al. 2013c) in order to assess the sensitivity of the downward thrust on the vehicle (the performance function) to changes in the side-vent-channel geometrical parameters.

Examples of the results yielded by the parametric study carried out in Ref. (Grujicic et al. 2013c) are depicted in Figures 3(a)–(b). Within this parametric study, circular cross-section channel-inlet area and the channel outlet-to-inlet area ratio were used as design variables, while the percent reduction in: (a) the total momentum transferred to the SBS, Figure 3(a); and (b) the maximum acceleration acquired by the SBS, Figure 3(b), (relative to the V-hull SBS case without side-vent-channels) were treated as the performance/objective functions (to be maximized).
Fig. 3. Percent reduction (relative to the SBS case without side-vent-channels) in: (a) total detonation-induced momentum transferred to the SBS; and (b) maximum SBS acceleration for the SBS configuration containing flared side-vent-channels, as a function of channel inlet-area and inlet-to-outlet area ratio Grujicic et al (2013c). The results are obtained using a combined finite-element/discrete-particle computational analysis. (c) The associated percent increase in the SBS mass in the same SBS design space.

The results displayed in Figure 3(a) reveal that: (a) there was a region in the channel inlet area/channel outlet-to-inlet area ratio design space which was associated with positive reductions in the blast momentum transferred to the SBS; (b) however, the maximum transferred-momentum reduction (ca. 3.5%) was relatively small. It should be noted that, due to the changing geometry and size of the side-vent-channels, the SBS mass was not constant throughout the design space used in Ref. (Grujicic et al. 2013c). Rather, the SBS mass increased both with the channel inlet area and outlet-to-inlet area ratio. Variation of the SBS mass throughout the design space is depicted in Figure 3(c) using a contour plot. A comparison of the results displayed in Figure 3(a) with those displayed in Figure 3(c) reveals that in the portion of the design space in which the transmitted impulse took on the lowest values, the SBS mass was ca. 2.2% larger than that of the V-hulled SBS without side-
vent-channels. This finding is important since an increase in the SBS mass (without any change in the SBS configuration) gave rise to an increase in the magnitude of the transferred momentum. This increase, for the estimated mass-range of the ejected soil interacting with the SBS, was assessed as ca. 0.4%. Based on this finding, it was concluded in Ref. (Grujicic et al. 2013c) that if the changes in the SBS configuration involving the introduction of side-vent-channels and their flaring could be accomplished without an increase in the SBS mass (e.g. through the use of light-armor materials with an increased blast efficiency), further reductions in the momentum transferred to the SBS could be expected.

The results displayed in Figure 3(b) reveal that: (a) the largest reduction in the maximum SBS acceleration was obtained in the portion of the design space in which both the channel inlet area and channel outlet-to-inlet area-ratio acquired the largest values. This finding is consistent with the fact that: (i) the SBS mass also acquired the largest values in this portion of the design space; and (ii) the SBS acquires a maximum acceleration during the earlier inertia-controlled stages of the blast/SBS interaction; and (b) however, as in the case of the momentum transfer, the largest reductions in the SBS maximum acceleration were found to be relatively small (ca. 3.3%). It was recognized in Ref. (Grujicic et al. 2013c) that a mere increase in the SBS mass resulted in an acceleration reduction. Since SBS configuration modifications involving the introduction of side-vent-channels and their flaring were accompanied by mass increases, the contributions of the increased mass and the lowered transferred momentum to the observed reductions in the SBS acceleration were separated in Ref. (Grujicic et al. 2013c). This was accomplished by combining the results displayed in Figure 3(b) with the results displayed in Figure 3(c), in order to first assess the portion of the percentage reduction in the maximum SBS acceleration which was the result of the associated mass increase. This procedure revealed that the SBS-mass increase reduced the SBS maximum acceleration by a maximum amount of ca. 2.9%. This finding was found to be consistent with the fact that the maximum SBS-mass increase due to the introduction of flaring of the side-vent-channels was ca. 2.9%. The (significantly smaller) remainder in the maximum acceleration reduction (3.3% – 2.9% = 0.4%) was attributed to the reduced momentum transferred to the SBS structure resulting from the introduction and flaring of the side-vent-channels. It should be recognized that SBS maximum acceleration is a key blast-variable which must be monitored/controlled since it is believed to correlate with the extent of occupants’ injury. Consequently, it was concluded in Ref. (Grujicic et al. 2013c) that further reductions in the SBS maximum acceleration were needed through the side-vent-channel concept refinement and solution optimization.

A first attempt to optimize the design of the side-vent-channel solution for maximum blast-mitigation performance was conducted by Grujicic et al. (2013d). The same combined finite-element/discrete-particle computational methods and tools as the ones used by Grujicic et al. (2013c) were utilized by Grujicic et al. (2013d). In addition, genetic algorithm was used to carry out side-vent-channel size/shape optimization. Within this optimization procedure, the following design variables were used: (a) the side-vent-channel inlet area; (b) the outlet-to-inlet area ratio; (c) the fraction of the channel length along which flaring is carried out; and (d) the major-to-minor-axes ratio at the side-vent-channel elliptically-shaped inlet section. As far as the objective function is concerned, it was defined as a weighted average of the percent reductions in the detonation-induced momentum transferred to, and the maximum acceleration acquired by, the SBS. The two weighing factors used in the definition of the objective function were not independent, but rather summed to 1.0. Thus, only one weighing factor, the weighing factor for transferred momentum, had to be defined and its
value varied between 1.0 (the optimization case in which only the minimization of the momentum transferred is considered) and 0.0 (the case in which the optimization is carried out only with respect to minimization of the maximum acceleration acquired by the SBS).

Fig. 4. Variation in the optimized values of the: (a) percent momentum reduction; and (b) percent maximum acceleration reduction (the two quantities whose weighted sum constitutes the objective function) with changes in the momentum-reduction weighing factor. The reference configuration corresponds to the V-hulled SBS without side-vent-channels. The associated percent change in the SBS mass is depicted in (c).

An example of the results yielded by the side-vent-channel shape/size optimization procedure conducted by Grujicic et al. (2013d), is shown in Figures 4(a)–(c). In these figures, the x-axis is associated with the transferred-momentum-reduction weighing factor. Variation of the momentum percent reduction and the acceleration percent reduction (relative to the case of the V-hulled SBS configuration without side-vent-channels) with the momentum-reduction weighing factor is depicted in Figures 4(a)–(b). The corresponding percent change in the SBS mass (also relative to
the case of the V-hulled SBS configuration without side-vent-channels) is depicted in Figure 4(c). Examination of the results depicted in Figures 4(a)–(b) reveals that, as expected: (a) at low values of the momentum-reduction weighing factor (i.e. in the acceleration-reduction-controlled regime), the percent reduction in the SBS acceleration is maximized, through the selection of the SBS shape and size parameters, while the transferred momentum is allowed to be compromised. Examination of Figure 4(c) shows that the optimal SBS configuration associated with the acceleration-reduction-controlled regime possesses the largest SBS mass, again as expected; and (b) at large values of the momentum-reduction weighing factor (i.e. in the momentum-reduction-controlled regime), the transferred-momentum reduction is maximized at the expense of the reduction in the SBS acceleration. Examination of Figure 4(c) shows that the optimal SBS configuration associated with the momentum-reduction-controlled regime possesses the smallest SBS mass, again as expected. Overall, the computational/optimization analysis carried out by Grujicic et al. (2013d) revealed that only minor additional improvements in the blast-mitigation performance of the side-vent-channel solution can be achieved through their shaping and sizing. Furthermore, questions were raised by Grujicic et al. (2013d) as to what effect the identified limitations of the combined finite-element/discrete-particle analysis have on the fidelity/accuracy of the computed reductions in the momentum transferred to, and the acceleration acquired by, an SBS equipped with side-vent channels.

Deficiencies of the computational analyses employed in Grujicic et al. (2013b,c,d) were addressed in Grujicic et al. (2015), in which a combined finite-element/discrete-particle/fluid-continuum computational approach was developed and utilized. Within this approach, the SBS was modeled as a finite-element-based structure, soil granules were treated as discrete particles, while air and gaseous detonation products were treated as continuum-type fluids. In addition, the problem of side-vent-channel size/shape optimization for maximum reduction of the transferred momentum and the maximum SBS acceleration was revisited and expanded. The expansion included the introduction of additional design variables related to the shape of the V-hull and the angle at which the lower portion of the side-vent-channel is cut relative to the ground. In addition, rather than combining the two objectives, i.e. the maximum reduction in: (a) the momentum transferred to; and (b) the maximum acceleration acquired by the targeted vehicle, into a single compound-objective function, a true bi-objective engineering-optimization analysis was employed in Grujicic et al. (2015). Due to conflicting nature of the two objectives, a set of the Pareto designs was identified, each associated with the optimal levels of the trade-off between the two objectives (at a given level of one of the objectives). An example of the bi-objective optimization results obtained in Grujicic et al. (2015) is depicted in Figure 5. This figure shows the variations in percent momentum reduction and percent acceleration reduction (both relative to their counterparts obtained in the case of the SBS fitted with a standard V-hull but no side-vent-channels) along the Pareto front. The results depicted in this figure clearly revealed the aforementioned trade-off between the two objectives. Points labeled A and B in Figure 5 correspond to the single-objective optimization solutions associated with the maximum reduction in momentum, and maximum reduction in maximum acceleration, respectively. As far as the improvements in the fidelity/accuracy of the solutions obtained using the newly-developed combined finite-element/discrete-particle/fluid-continuum computational approach is concerned, it was established that, for the most part, the simpler and computationally more efficient combined finite-element/discrete-particle approach yields the results quite comparable to the ones obtained using the approach proposed in Grujicic et al. (2015).
1.3. Main Objectives

In the computational and optimization analyses of the side-vent-channel solution carried out in Grujicic et al. (2013b,c), the side-vent-channels were assumed to be either rigid or to be made of a single (elastic) armor-grade steel material. The present work will investigate the effect of the side-vent-channel material selection on the blast-mitigation efficiency of this solution. Specifically, the present work has the following two main objectives: (a) to examine the ability of sandwich-structures consisting of steel face-sheets and an aluminum-foam core to improve blast-mitigation efficiency of this solution; and (b) to optimize the aluminum-foam core-density profile for maximum reduction of the transferred momentum and the maximum SBS acceleration.

1.4. Paper Organization

A brief description of the problem analyzed in the present work and a brief overview of the computational methods and tools used to analyze the interaction of the air-blast-ejected soil and detonation products with the target SBS are provided in Section 2. The main results obtained in the current work are presented and discussed in Section 3. The key conclusions yielded by the present work are summarized in Section 4.

![Graph](image)

**Fig. 5.** Variations of the two components of the objective function along the Pareto front. Please see text for definition of points A and B.

2. Problem Description and Computational Analysis

2.1 Formulation of the Problem

The basic problem analyzed in the present work involves detonation of a mine shallow-buried underneath the targeted SBS (equipped with the V-shaped hull, with side-vent-channels), and the
subsequent interaction of gaseous detonation products, soil ejecta and blast waves with the SBS underbody. As mentioned earlier, for improved blast-mitigation capacity, all-steel side-vent-channels are replaced with their sandwich-structure counterparts. Details regarding the translation of this physical problem into the corresponding mathematical model and the computational techniques used to investigate this problem and to establish blast-mitigation potential of the “modified/upgraded” side-vent-channel solution are presented in the following sections.

2.2 Computational Domain

The computational domain used, Figure 6, consists of three distinct subdomains: (a) a continuum-structure Lagrangian-type; (b) a dispersed discrete-particle Lagrangian-type; and (c) a fluid-continuum Eulerian-type.

2.2.1 Continuum-Structure Subdomain

The continuum-structure subdomain is associated with the SBS (equipped with the V-hull, with side-vent-channels) and modeled using three-noded shell elements. Depending on the SBS configuration modeled, the number of finite elements ranged between ca. 60,000 and 80,000. To reduce the computational cost, the SBS and its V-hull are modeled as rigid (single armor-grade steel) structures. On the other hand, the side-vent-channels are modeled as deformable structures made of either: (a) a single armor-grade steel (the reference case); or (b) sandwich-structures consisting of steel face-sheets and an aluminum-foam core. To model sandwich-structures,
“composite shell elements” are used within which one can define the thicknesses, materials and the stacking order of different shell layers/plies. It should be noted that the computational domain depicted in Figure 6 and used in the present work was obtained as a result of the SBS shape/size optimization procedure carried out in Grujicic et al. (2015). Close examination of Figure 6 reveals that the geometry of the continuum-structure subdomain was modified in this optimization procedure, relative to the geometry displayed in Figure 2. Specifically, in place of a constant-angle V-hull, a two-angle V-hull design is introduced for improved blast-mitigation performance. Additional modifications made in the SBS geometrical model concern the use of added masses of proper magnitude and placement to account for the components such as engine, drive-train, wheels, turret, cabin interior including the occupants, etc. This was done in order to improve the fidelity of the SBS model with respect to the total mass and the overall moment of inertia.

2.2.2 Discrete-Particle Subdomain

As far as the discrete-particle subdomain is concerned, it comprises soil particles (mine-casing fragments are not modeled in this computational-investigation of the side-vent-channel blast-mitigation efficacy). The discrete-particle subdomain possesses a rectangular parallelepiped external shape and is placed at the appropriate stand-off distance (SOD) from the SBS. Except for a circular-disk-shaped region placed at the appropriate DOB, and filled with the mine (or more precisely, high-pressure gaseous detonation products), the domain is filled randomly with the soil particles to achieve the desired packing-density of the soil. Following our prior work (Grujicic et al. 2013c), in which the effect of the number of soil particles (or, alternatively, the size of the soil super-particles), was investigated, the number of discrete particles used was ca. 140,000–160,000.

2.2.3 Continuum-Fluid Subdomain

The fluid-continuum Eulerian subdomain is of a rectangular-parallelepiped geometry, and initially envelops the other two subdomains. It is meshed using cubic-shaped cells. After conducting a preliminary cell-size sensitivity analysis, the number of Eulerian cells was chosen to be in the range of ca. 600,000–800,000.

2.2.4 Initial Configurations

Initial configurations of the three computational subdomains utilized in the present work are depicted in Figure 6. It should be noted that the SBS structure possesses two vertical planes of symmetry, suggesting that only one-quarter of the computational domain needs to be analyzed explicitly. However, the soil bed contains a random distribution of the particles and, thus, does not possess the same two planes of symmetry. Consequently, the distribution of the velocities of ejected soil particles is generally asymmetric, and the entire computational domain had to be analyzed. By analyzing the entire computational domain, rather than one of its quarters, potential lateral translation and rotation of the SBS are also enabled.

2.3 Combined Finite-Element/Discrete-Particle/Fluid-Continuum Approach

As mentioned earlier, the computational approach used in the present work is the same as that utilized in Grujicic et al. (2015). Within this approach: (a) the SBS (with V-hull and side-vent-
channels) is modeled as a (either deformable or non-deformable) finite-element continuum structure; (b) soil ejecta are modeled as discrete particles; while (c) the gaseous detonation products and air are modeled as a fluid continuum. To completely define the computational model of the problem under investigation, the following functional relationships must be defined: (i) governing conservation equations; (ii) functional relations controlling interactions between materials of the same (e.g. soil granules) and different (e.g. gaseous detonation-products and soil-particles) types; (iii) material-specific constitutive relations defining the mechanical behavior of the attendant materials under different kinematic and kinetic conditions; and (iv) other auxiliary relations such as the initial and the boundary conditions. Since details regarding these functional relationships can be found in Grujicic et al. (2015), only a brief overview of the same will be provided in the remainder of this section.

2.3.1 Structural Finite-Element Formulation

As mentioned earlier, the SBS and V-hull are modeled as rigid structures, while the side-vent-channels are modeled as deformable structures. For the deformable structures, the conventional displacement-/stiffness-based finite-element formulation was employed. Within this formulation, the subject (continuum-type) structure is first discretized into a number of finite-size interconnected elements. Next, the governing mass and momentum conservation equations for the structure are recast at the level of individual elements, combined with the associated material constitutive relations and initial/boundary/loading-conditions, integrated out, and assembled into a system of algebraic equations. The system of equations is next solved using one of the matrix-type linear/nonlinear solution algorithms in order to determine the mechanical/dynamic response of the structure subjected to the prescribed loading and kinematic constraints. Since the basic matrix-type equations for the finite-element method described here can be found in many texts and published papers, e.g. Grujicic and Bell (2011b), these equations will not be presented in this manuscript.

For the rigid structures, the following has to be specified: (a) mass; (b) location of the center of mass (COM); and (c) the moment of inertia second order tensor relative to the COM. These quantities are determined from the knowledge of the mass, COM and interconnectivity of individual elements through the application of standard functional relations for an assembly of discrete particles (finite-elements, in the present case). Since these relations can be found in many texts and published papers (e.g. Grujicic et al. (2010a)), they will not be repeated here.

2.3.2 Discrete-Particle Formulation

Within this formulation, each attendant material (soil granules, in the present work) is represented as an assembly of discrete, rigid, spherical, interacting particles which exchange momentum and kinetic energy during their collisions/contact with each other and with the finite-element-based SBS and the continuum-fluid-based gaseous detonation-products and air. Typically, to make the computational cost manageable, groups of individual particles are first clumped into “super-particles” (referred to simply as particles hereafter) and the analysis is carried out using the latter. Within this formulation, particle motion involves both translation and rotation, and is governed by the force-based and moment-based Newton’s second law (Grujicic et al. 2013c). Due to its
Lagrangian character, this formulation has a number of advantages in comparison to the Eulerian-based formulations (Grujicic et al. 2013c).

2.3.3 Continuum-Fluid Formulation

Within this formulation, the behavior of the fluid material(s) (gaseous detonation products and air, in the present case) is governed by the continuity and Navier-Stokes (linear-momentum conservation) equations. However, in order to account for the fact that a portion of the space containing the fluid materials is occupied by the discrete-particle (and, perhaps, the structural-continuum) materials, and the fact that the fluid may be a mixture of materials, the locally-averaged continuity and Navier-Stokes equations for (incompressible) flow through a porous medium are normally used (Grujicic et al. 2015).

2.3.4 Interactions

In general, within the computational analyses such as the one carried out in the present work, provisions must be made for interactions between the same, same type, and different types of materials. The number of interaction types is greatly reduced in the present case due to the fact that there is one structural continuum material (SBS), one discrete-particle material (soil), and a two-material fluid mixture (consisting of gaseous detonation products and air). In fact, neglecting some of the secondary details related to the nature of the contacting materials, only two basic types of interactions are identified in the present work: (a) particle/particle and particle/SBS interactions; and (b) fluid/particle and fluid/SBS interactions. These two types of interactions are briefly overviewed below.

Particle/Particle and Particle/SBS Interactions

While the description presented below pertains explicitly to the case of particle/particle interactions, it is equally applicable to the particle/SBS interactions. Particle/particle interactions are modeled in the present work using the so-called penalty contact algorithm Grujicic and Bell (2011b) rather than the computationally-efficient (but physically over-simplified) elastic (or, more precisely, kinematic-elastic) collision algorithm. This was done in order to account for the effects such as: (i) soil-particle finite stiffness; (ii) rate-dependent dissipative/damping nature of the inter-particle collisions; and (iii) the inter-particle frictional effects.

The essential features of the particle/particle and particle/solid-structure penalty-contact algorithm used can be found in Grujicic et al. (2013c). In the same reference, procedures are presented for the parameterization of this algorithm.

Fluid/Particle and Fluid/SBS Interactions

Determination and quantification of the interactions between the Eulerian fluids and Lagrangian solids is often quite a challenging problem, due to the fact that the associated subdomains generally do not possess conformal meshes. This problem is compounded in cases in which the size of discrete particles is relatively small in comparison to the Eulerian-mesh cell size. In such cases, various approximation/simplification schemes such as the so-called "two fluid model" (Gidaspow 1994, Grujicic et al. 2007) and “computational fluid dynamics - discrete element method (CFD-DEM)”
(Tsuji et al. 1992) are used. Within these schemes, different approximations/simplifications are utilized in order to account for the presence of the discrete particles within the fluid, without explicitly representing/modeling these particles. Fortunately, in the present case, the size of the discrete particles (i.e. soil granules) is comparable to that of the Eulerian-mesh cell, and these oversimplified algorithms do not have to be used. Instead, the so-called “immersed boundary method” (Grujicic et al. 2012a) is employed in the present work in order to model fluid/particle and fluid/SBS interactions. Details regarding the formulation and parameterization of the immersed boundary method can be found in Grujicic et al. (2015).

2.3.5 Material Models

Since the present model consists of three computational subdomains, separate constitutive models are described, in the remainder of this section, for all the materials residing within the subdomains.

**Lagrangian SBS Subdomain**

**Rigid-Steel Material Model**: For the rigid portions of the Lagrangian SBS subdomain, the only material parameter which had to be defined was the mass density (set to 7850 kg/m³, a typical value for steel).

**Deformable-Steel Material Model**: As mentioned earlier, the side-vent-channels are treated as deformable structures and, hence, more comprehensive material constitutive models had to be defined. For the reference case, associated with single armor-grade (AISI 4340) steel side-vent-channels, the material constitutive behavior is represented using the conventional Johnson-Cook material model (Grujicic et al. 2010b). A detailed overview of this material model, including its AISI 4340 parameterization, can be found in our recent work (Grujicic et al. 2008, 2010b). In the case of sandwich-structure side-vent-channels, the face-sheets are assumed to be made of AISI 4340, and hence are represented using the same material model. On the other hand, the constitutive behavior of the aluminum-foam core is assumed to be consistent with the so-called “crushable foam” material model with volumetric hardening. A brief description of this model is given in the remainder of this subsection.

**Crushable-Foam Material Model**: The volumetric-hardening crushable-foam material model tries to capture the following experimental observations pertaining to the deformation behavior of metallic foams: (a) the material responds differently to the applied tension and compression; (b) under compression, material deformability is enhanced by cell-wall buckling processes (Gibson and Ashby 1982, Grujicic et al. 2011a); (c) since foam deformation is not associated with an instantaneous removal upon unloading, it can be considered to be of an inelastic/plastic character, at least for short-duration events (such as blast-loading of an SBS); (d) load-bearing capacity is substantially lower in tension than in compression; and (e) under pure shear or negative hydrostatic-pressure stress states, foams exhibit very little strain-hardening, while under positive hydrostatic-pressure stress states, substantial strain-hardening effects are generally observed.

The elastic portion of the crushable-foam material model is represented using the conventional generalized Hooke’s Law. Within the isotropic linear-elastic approximation adopted in the present work, the fourth-order elastic stiffness tensor relating the second-order stress tensor with the
second-order elastic-strain tensor contains only two (density-dependent) elastic moduli (typically chosen as the Young's modulus and Poisson's ratio). As far as the plastic response of the crushable foam is concerned, its full definition requires specification of: (a) yield criterion/surface; (b) flow-potential/rule; and (c) a strain-hardening law. Within the volumetric-hardening crushable-foam material model, the yield surface is defined as:

\[ F = \sqrt{q^2 + \alpha^2(p - p_0)^2} - B = 0 \]  

where the von Mises equivalent stress \( q = \sqrt{\frac{3}{2}S : S} \), hydrostatic pressure \( p = -\frac{1}{3}\text{trace}\sigma = -\frac{1}{3}\sigma : I \), \( S \) is the stress deviator and \( \sigma \) is the total stress. In the meridional \( p-q \) plane, the yield surface is represented by a half-ellipse centered at \((p = p_0, q = 0)\). There are three parameters defining the yield surface: (a) \( p_0 \); (b) \( \alpha \), a yield-surface shape factor; and (c) \( B = g(p = p_0) \), the length of the (vertical) \( q \)-axis of the yield ellipse. These parameters, as explained below, are normally defined in terms of the: (a) (positive) uniaxial-compressive material strength, \( \sigma_c \); (b) (positive) hydrostatic-compressive material strength, \( p_c \); and (c) (also positive) hydrostatic-tensile material strength, \( p_t \).

On the geometrical grounds, \( p_0 \) can be defined as \( p_0 = (p_c - p_t)/2 \). \( \alpha \) and \( B \) are defined by substituting the yield conditions into Eq. (1) for: (a) uniaxial compression \(- q = \sigma_c \) and \( p = \sigma_c/3 \); and (b) hydrostatic compression \(- q = 0 \) and \( p = p_c \), and by solving the resulting system of two algebraic equations to get:

\[ \alpha = \frac{3k}{\sqrt{(3k_t + k)(3-k)}} \]  

\[ B = \alpha A = \alpha \frac{p_c + p_t}{2} \]  

where \( k = \sigma_c^0/p_c^0 \) and \( k_t = p_t/p_c^0 \), and superscript 0 is used to denote the initial value of the respective quantities.

Examples of the initial yield surface and two plastic-deformation-modified yield surfaces are depicted in Figure 7(a). The initial yield surface, labeled as '0' in Figure 7(a), is defined for the following set of parameters: \( \sigma_c = 10 \text{ MPa}, k = 0.75, \text{ and } k_t = 0.5 \).

The flow potential within the volumetric-hardening crushable-foam material model is defined as:
\[ G = \sqrt{q^2 + \frac{9}{2} p^2} \] (4)

An example of a set of iso-flow potential lines in the \( p - q \) plane is depicted in Figure 7(b).

**Fig. 7.** (a) An example of an initial (labeled ‘0’) and two plastic-deformation-modified (labeled ‘1’ and ‘2’) yield surfaces for a volumetric-hardening crushable-foam; and (b) a set of the corresponding iso-flow-potential lines.

Within the volumetric-hardening crushable-foam material model, plastic flow (as quantified by the plastic strain rate tensor, \( \dot{\varepsilon}^{pl} \)) is assumed to be governed by the following flow rule:

\[ \dot{\varepsilon}^{pl} = \frac{\dot{\varepsilon}^{pl} \partial G}{\partial \sigma} \] (5)

where the equivalent plastic strain rate \( \dot{\varepsilon}^{pl} \) is defined as:

\[ \dot{\varepsilon}^{pl} = \frac{\sigma : \dot{\varepsilon}^{pl}}{q} \] (6)

It should be noted that the functional form of Eq. (4) predicts that the plastic-flow direction is collinear with the stress direction for the cases of monotonic (radial) loading paths. This is consistent with the experimental observations, which suggest that loading along a single principal direction gives rise to insignificant extent of plastic flow in the remaining principal directions. Furthermore, it should be recognized that since the separate functional relations are defined for the yield surface and for the plastic potential, the plastic flow in the present case is of a non-associative character, and involves a non-symmetric system of governing equations.
As plastic deformation of the foam proceeds, its yield surface evolves while maintaining its semi-elliptical shape. It is generally assumed that: (a) \( p_t \) does not change in the course of plastic deformation; (b) \( p_c \) increases/decreases with an increase/decrease in the foam density (i.e. positive volumetric-compressive strain, \( \varepsilon_{\text{vol}}^{\text{pl}} \)); and (c) due to the zero effective plastic Poisson’s ratio, the volumetric-compressive strain is numerically equal to the uniaxial-compressive strain (\( \varepsilon_{\text{axial}}^{\text{pl}} \)). Taking this into account, and assuming that the yield-surface shape factor \( \alpha \) remains constant, the following evolution equation for \( p_c \) can be derived:

\[
p_c \left( \varepsilon_{\text{vol}}^{\text{pl}} \right) = \frac{\sigma_c \left( \varepsilon_{\text{axial}}^{\text{pl}} \right) \left[ \sigma_c \left( \varepsilon_{\text{axial}}^{\text{pl}} \right) \left( \frac{1}{\sqrt{3}} + \frac{1}{9} \right) + \frac{p_t}{3} \right]}{p_t + \frac{\sigma_c \left( \varepsilon_{\text{axial}}^{\text{pl}} \right)}{3}} \tag{7}
\]

Eq. (7) suggests that the evolution of yield surface during plastic deformation is fully defined by the \( \sigma_c \) vs. \( \varepsilon_{\text{axial}}^{\text{pl}} \) relationship. The two plastic-deformation-modified yield surfaces (labeled ‘1’ and ‘2’) in Figure 7(a) were obtained under the following strain-hardening conditions: \( \sigma_c = 14 \) MPa at \( \varepsilon_{\text{axial}}^{\text{pl}} = 0.25 \), and \( \sigma_c = 20 \) MPa at \( \varepsilon_{\text{axial}}^{\text{pl}} = 0.5 \), respectively.

The hardening behavior for a family of Al-foams used in the present work is depicted as a contour plot in Figure 8. Contour levels in this figure correspond to the uniaxial compressive strengths in MPa. The two stress ratios for all the members of this family are assumed to be constant: \( k = 0.75 \) and \( k_t = 0.5 \).

**Fig. 8.** Hardening behavior for the family of Al-foams used in the present work. Contour levels correspond to the uniaxial compressive strengths in MPa.
Lagrangian Discrete-Particle Subdomain

Within the present formulation, soil particles are explicitly considered as being rigid (i.e. non-deformable). However, the effective stiffness of the soil particles is not infinite but rather of a finite value. The reason for this is that, as discussed above, particle/particle and particle/SBS interactions are assumed to be of a non-rigid elastic type, but rather to be associated with finite values of the normal and shear contact-spring stiffnesses. In other words, the (non-rigid) constitutive response of the soil particles is accounted for through the use of the Hertz contact solution between two elastic spheres or a semi-infinite planar domain and a sphere, as implemented in the non-rigid penalty-contact algorithm.

Eulerian-Fluid Subdomain

This subdomain contains a mixture of two fluids, i.e. air and gaseous detonation products. For fluids, the material constitutive behavior is described by: (a) an equation of state, EOS (a functional relationship between pressure, mass-density/specific-volume and internal-energy-density/temperature); and (b) a functional relationship between the shear stress and the shear strain and strain-rate. Two types of EOS were utilized in the present work: (i) ideal gas EOS; and (ii) Jones-Wilkins-Lee (JWL) EOS (ANSYS 2009). These two types of EOS are typically selected in the analysis of various detonation/explosion scenarios. Since a detailed overview of the ideal-gas and JWL EOS relationships for the air/detonation-products mixture and their parameterization for the case of C-4 high explosive (HE), the explosive utilized in the present work, can be found in our prior work (Grujicic et al. 2012b,c, 2013a), the same details will not be repeated here.

Since the air/detonation-products mixture is gaseous, it has a zero effective shear stiffness. Thus, shear strain within this material does not generate shear stress. However, shear stresses can be developed as a result of a gradient in the flow velocity, i.e. shear-strain-rate. In the present work, the Newtonian-fluid model is adopted for the gaseous mixture, within which the shear stress is assumed to scale linearly with the velocity gradient (with the proportionality constant, the viscosity, being set at 1.78·10^{-5} Pa·s).

It should be noted that, within the current formulation, the initially unexploded mine made of C-4, a solid material, is not modeled explicitly. Rather, detonation is treated as an instantaneous process which converts, at the beginning of the computational analysis, unreacted explosive into high-pressure, high-temperature gaseous detonation products. This is the reason why no material model had to be defined for the (solid) C-4 HE.

2.3.6. Computational Analysis-Type

The mine blast event and the subsequent interactions between the detonation products, soil ejecta and air blasts and the SBS are analyzed computationally using the combined finite-element/discrete-particle/fluid-continuum algorithm. Due to the extremely short duration of the mine-blast detonation event (ca. tens of milliseconds), heat transfer between the Eulerian and Lagrangian subdomains, as well as heat transfer by conduction within the fluid subdomain, are not considered. In other words, only convective heat transfer within the fluid is considered.
2.3.7 Initial Conditions

Prior to the beginning of the computational analysis, the Eulerian subdomain is filled with atmospheric-pressure/room-temperature air except for the “mine region,” which is filled with high-pressure, high-temperature gaseous detonation products. Then the soil-discrete-particle subdomain is placed in the lower portion of the Eulerian subdomain. Filling of the former subdomain with soil particles, to achieve the desired packing density, is carried out in accordance with the procedure described in our prior work (Grujicic et al. 2013c). This procedure ensures that no “inter-particle penetration” (except the one caused by gravity) exists between the neighboring soil particles. The particles are assumed to initially be at rest. Lastly, the continuum-Lagrangian SBS subdomain is placed in the upper portion of the Eulerian subdomain at the desired SOD. The SBS subdomain is assumed to initially be stationary and, in its non-rigid portions, to be stress-free.

2.3.8 Boundary Conditions

The Eulerian subdomain is subjected, over its external faces, to the no-inflow and (101.3 kPa external pressure) free-outflow boundary conditions. The discrete-particle subdomain is subjected, over its bottom and lateral four faces, to the non-reflecting outflow boundary conditions (in order to avoid unphysical reflection of the shock waves from these boundaries). On the other hand, no displacement/stress boundary conditions are applied over the top face of the discrete-particle subdomain. Except for loading due to gravity, no boundary conditions are applied to the SBS subdomain.

2.3.9 Computational Algorithm

The computational model described in the preceding sections is analyzed numerically using the so-called CEL (Combined Eulerian-Lagrangian) algorithm, as implemented in ABAQUS/Explicit (Dassault Systemès 2010), a general-purpose finite-element solver. This algorithm enables an analysis of the interactions between Eulerian and Lagrangian subdomains. Within the continuum-type Lagrangian subdomains (SBS, in the present case): (a) the mesh (nodes and elements) is attached to the associated material and moves and deforms with it; and (b) each layer/ply of an element must be fully filled with a single material. On the other hand, within an Eulerian subdomain: (a) the mesh is fixed in space and the material flows through it; (b) elements/cells are allowed to be partially filled and/or contain multiple materials; and (c) since the material and the element boundaries do not generally coincide, a separate (“interface reconstruction”) algorithm must be used to track the position of Eulerian material boundaries. The interface reconstruction algorithm approximates the material boundaries within an element as simple planar facets and, hence, accurate determination of a material’s location within an element requires the use of fine Eulerian meshes. Since, within the present formulation, the two Eulerian materials (air and gaseous detonation products) are assumed to be mixed at the molecular level, Eulerian-material boundaries did not need to be tracked, and relatively larger Eulerian cells could be used (making the computational analysis more efficient).

Typically, the following sequence of computational steps is carried out within each time increment: (a) Eulerian/Lagrangian interaction algorithm is used to determine loading experienced by the Lagrangian subdomain(s); (b) conventional displacement-based finite-element analysis is carried
out to update the deformation state and the position of the Lagrangian domains; (c) the current position of the Lagrangian domains is used to redefine the fluid-filled Eulerian subdomain; and (d) governing equations for the fluid are then solved as a two-step process: (i) within the so-called Lagrangian substep, the Eulerian subdomain is temporarily treated as being of a Lagrangian character (i.e. its nodes and elements are attached to and forced to move with the underlying material); and (ii) within the so-called "remap" step, the distorted Eulerian mesh is mapped onto the original Eulerian mesh and the accompanying material transport is computed and used to update the Eulerian-material states and inter-material boundaries (when required).

2.3.10 Computational Accuracy, Stability and Cost

A standard particle-size and mesh-refinement sensitivity analysis was carried out (the results not shown for brevity) in order to ensure that a convergence of the key results is reached with respect to the further refinement of these geometrical/mesh parameters. Due to the conditionally-stable nature of the explicit finite element analysis used, the maximum time increment during each computational step had to be kept lower than the attendant stable time increment. A typical 50 ms computational analysis followed by a detailed post-processing data-reduction procedure required on average 2 hours of (wall-clock) time on a 12-core, 3.0 GHz machine with 16 GB of memory.

3. Results and Discussion

In this section, the key results obtained in this work are presented and discussed. First, some prototypical results yielded by the employed three-dimensional combined finite-element/discrete-particle/fluid-continuum computational model and analysis, for the cases of the side-vent-channels being made from either a single armor-grade steel or a sandwich-structure consisting of steel face-sheets and aluminum foam-core, are overviewed and examined. The two basic architectures of the side-vent-channel walls are shown in Figures 9(a)–(c). Figure 9(a) depicts a short section of the cylindrical side-vent-channel (modeled as a shell component). A small curved sector of the shell is marked yellow in this figure. Next, the same sector is depicted as an all-steel and sandwich-structure three-dimensional component in Figures 9(b)–(c), respectively. When discussing these results, particular attention is paid to providing insight into the ability of the modified V-shaped hull and the side-vent-channels to lower the blast momentum transferred to, and the maximum associated acceleration acquired by, the SBS through the operation of venting and downward-thrust effects. In the second portion of this section, the results of an optimization procedure employed to identify an optimal architecture of the sandwich-structure (as defined by the face-sheet/foam-core thicknesses and the foam-density variation through the core) are presented and critically assessed. Two types of aluminum-foam density variation through the core are considered: (a) a linear type, Figure 10(a); and (b) a balanced bi-linear type, Figure 10(b). In both cases the density variation is characterized by two parameters: (a) the mean value of density; and (b) the rate of change of density through the core thickness at one (inward-facing or outward-facing) of the core faces.
3.1 Prototypical Results

The employed computational analysis yielded the results pertaining to the temporal evolution and spatial distribution of various particle-state and continuum-field quantities such as particles’ position, (translational and rotational) particle velocities, particle/particle and particle/continuum-structure interaction forces and moments, etc. In addition, results pertaining to the explosive-charge detonation-induced loading experienced by, and the subsequent response of, the SBS structure were obtained. In the remainder of this section, a few prototypical results are presented and discussed.

Fig. 9. (a) A short section of the cylindrical side-vent-channel (modeled as a shell component); (b) and (c) the yellow curved sector in (a) depicted as an all-steel and sandwich-structure three-dimensional component, respectively.
Fig. 10. Two modes of variation of the aluminum-foam density through the core thickness: (a) linear; and (b) balanced bi-linear.

3.1.1. Temporal Evolution of the Attendant-Material Spatial Distribution

Spatial distribution of the Lagrangian attendant materials (i.e., steels used in the SBS and side-vent-channel construction, and soil) at four (1 ms, 1.5 ms, 2.5 ms, and 3.9 ms) post-detonation times in the case of the SBS equipped with all-steel (flared) side-vent-channels is shown in Figures 11(a)–(d). For improved clarity, in these figures as well as in Figures 12(a)–(d), only a fraction of the soil particles present in the model is shown and the size of the retained soil particles is increased. The corresponding results at the same four post-detonation times, but for the case of an SBS equipped with side-vent-channels made of sandwich-structures consisting of steel face-sheets and aluminum-foam core, are presented in Figures 12(a)–(d). Examination of the results displayed in Figures 11(a)–(d) and 12(a)–(d), and comparison of the results for the two side-vent-channel architectures at the same post-detonation times reveals:

(a) The presence of the modified V-hull with side-vent-channels helps guide the flow of the soil-ejecta along the direction parallel with the side of the V-hull. The same could be said for the gaseous detonation products and air [the results are not shown for brevity, but are quite similar to the ones displayed in Figures 10(a)–(d) in Grujicic et al (2015)];

(b) A counting analysis of the soil particles passing through the side-vent-channels revealed that the side-vent-channels closest to the mine play the most critical role in venting the mine-detonation-propelled soil particles;

(c) Furthermore, it is seen that the ejected soil initially retains its cohesion and relatively high density, Figures 11(a) and 12(a), while at later post-detonation times, ejected soil breaks up into non-bonded particles and acquires a low density, Figures 11(c)–(d) and 12(c)–(d); and

(d) The side-vent-channels made of sandwich-structures have sustained more permanent deformation than their all-steel counterparts, Figure 12(d) vs. Figure 11(d). Considering the fact that permanent deformation within sandwich-structures is accompanied by extensive compaction of its aluminum-foam core (a process associated with a considerable absorption/dissipation of the energy), one would expect reductions in the blast-loading experienced by, and the maximum...
acceleration acquired by the SBS. This prediction will be confirmed by the results displayed in Figures 13(a)–(b).

![Spatial distribution of the Lagrangian attendant materials](image)

**Fig. 11.** Spatial distribution of the Lagrangian attendant materials, in the case of the SBS equipped with all-steel (flared) side-vent-channels, at post-detonation times of: (a) 1 ms; (b) 1.5 ms; (c) 2.5 ms; and (d) 3.9 ms.

3.1.2. SBS Velocity and Acceleration Temporal Evolution

The results presented in the previous section suggested that the replacement of the all-steel side-vent-channels with their sandwich-structure counterparts may have a positive role in reducing the effect of buried-landmine detonation on the momentum transferred to, and the acceleration acquired by, the SBS. In this section, more quantitative results pertaining to the temporal evolution of the SBS velocity and acceleration are presented and discussed. Examples of the typical SBS ($z$-component-dominated, total translational) velocity vs. time and the corresponding SBS acceleration vs. time results obtained in the present work are shown in Figures 13(a)–(b), respectively. In the case of each of these figures, two sets of results are presented, one for the all-steel side-vent-channel architecture and the other for the sandwich-structure side-vent-channel architecture with linear variation of the aluminum-foam density through the core thickness. In the case of Figure 13(a), the velocity is normalized by the maximum velocity of the SBS equipped with the all-steel side-vent-channels while the time is normalized by its value corresponding to the SBS maximum velocity, also for the case of the SBS equipped with the all-steel side-vent-channels. Likewise, in the case of Figure 13(b), the acceleration is normalized by the maximum acceleration of the SBS equipped with the all-steel side-vent-channels while the time is normalized by its value corresponding to the SBS maximum velocity, also for the case of the SBS equipped with the all-steel side-vent-channels.
Fig. 12. Spatial distribution of the Lagrangian attendant materials, in the case of the SBS equipped with sandwich-structure (flared) side-vent-channels, at post-detonation times of: (a) 1 ms; (b) 1.5 ms; (c) 2.5 ms; and (d) 3.9 ms.

Fig. 13. Examples of the typical: (a) SBS normalized-velocity vs. time; and (b) SBS normalized-acceleration vs. time results. In each case, the results pertaining to the all-steel and sandwich-structure architecture of the side-vent-channels are depicted.
Examination of the results displayed in Figures 13(a)-(b) shows that, for both side-vent-channel architectures, the SBS velocity initially experiences a sharp rise, reaches a peak value and then gradually decreases under the influence of gravity. As far as acceleration is concerned, it reaches its peak value much earlier than the velocity and then sharply drops as the additional momentum transfer to the SBS from the detonation products and soil ejecta decreases. Furthermore, examination of the Figure 13(a) results for the two side-vent-channel architectures shows that there is a reduction in the maximum SBS velocity when the all-steel side-vent-channel configuration is replaced with its sandwich-structure counterpart. On the other hand, the same change in the side-vent-channel configuration does not effectively change the maximum acceleration experienced by the SBS, Figure 13(b).

3.2 Side-Vent-Channel Wall Sandwich-Structure Architecture Optimization

3.2.1. Optimization Procedure

In this section, the optimization results of the side-vent-channel wall sandwich-structure architecture are presented and discussed. The conventional Simplex optimization algorithm overviewed in Grujicic et al. (2015) has been utilized. A complete definition of an optimization problem requires specification of the following: (a) design-variables; (b) constraints and (c) the objective function (or functions, in the case of a multi-objective optimization problem, as is the present case).

**Design Variables**: The following four design variables are used in the present work: (a) steel face-sheet thickness, $DV_1$; (b) foam-core thickness, $DV_2$; (c) foam average density, $DV_3$; and (d) rate of change of the foam density with core thickness at the inward-facing side of the sandwich core, $DV_4$. The inward-facing side of the sandwich core is the side which faces the interior of the side-vent-channel.

**Constraints**: Optimization has been carried out under the following three constraints: (a) areal density of the sandwich-structure is equal to its all-steel counterpart; (b) the two steel face-sheets are of equal thicknesses; and (c) foam-density variation through the core thickness is of a linear or balanced bi-linear character.

**Objective Function(s)**: The two (conflicting) objectives of the present optimization-procedure are to: (i) maximize the reduction in the detonation-induced momentum transferred to the SBS; and (ii) maximize the reduction in the maximum acceleration acquired by the SBS. To ensure that the best possible trade-off between the two conflicting objectives is attained at a given level of either of the objectives, a true bi-objective optimization-procedure is employed. Within this approach, the objective function is treated as a two-component vector/array. The conflicting nature of the two objective-function components used in the present work can be understood by considering the effect of the SBS mass on the two. That is, as the SBS mass is increased/decreased, the momentum transferred to the SBS is also increased/decreased while the maximum acceleration acquired by the SBS is decreased/increased. It should be noted that due to the conflict between the two objective-function components, one cannot expect a unique solution to the optimization problem. Instead, the result of the bi-objective optimization will be a series of so-called “non-inferior” solutions forming the so-called “Pareto front.” The unique feature of the Pareto-front optimization solutions is that there are no infinitesimally small perturbations in the design variables associated with these solutions which could simultaneously improve both components of
the objective-function vector. In other words, such perturbations will always be associated with a trade-off between the two components of the objective-function. To clarify this point, a schematic is shown in Figure 14 of the two-component objective-function space. The feasible portion of this space is colored light green. In this figure, the Pareto front is represented by curve $A-B$, where points $A$ and $B$ correspond respectively to the maximum (optimal) values of the two objective-function components. In the interior portion of the objective-function feasibility space, one can always perturb the given design (as defined by the values of the two objective-function components) in order to simultaneously increase both objective-function components. No such perturbations are possible along the $A-B$ line (since they would place the design outside the feasible domain, the region colored yellow in Figure 14). For this reason, the designs associated with the Pareto front $A-B$ are generally referred to as being “non-inferior.”

**Fig. 14.** Variations of the two components of the objective function along the Pareto front. The two Pareto fronts correspond to the cases of all-steel and sandwich-structure side-vent-channel architectures.

### 3.2.2. Results

The results of the Simplex-algorithm-based bi-objective optimization procedure for the side-vent-channel wall sandwich-structure architecture are displayed in Figures 15 and 16(a)–(d). In each of these figures, the $x$-axis represents the first component of the objective function, i.e. the percent reduction in the transferred-momentum (relative to their counterparts in the case of the SBS fitted with a standard V-hull but no side-vent-channels) residing on the Pareto front.
Trade-off between Objective-function Components along the Pareto Front: Variation of the two components of the objective function, i.e. the momentum percent reduction and the acceleration percent reduction (relative to their counterparts in the case of the SBS fitted with a standard V-hull but no side-vent-channels) along the Pareto front is depicted in Figure 15. Three sets of results are presented in this figure, for the following side-vent-channel architectures: (a) all-steel; (b) sandwich-structure with linear aluminum density variation through the core; and (c) sandwich-structure with balanced bi-linear aluminum density variation through the core. The results for the all-steel side-vent-channel configuration are taken from Grujicic et al. (2015). Examination of the results depicted in Figure 15 reveals that, as expected, there is a trade-off between the two components of the objective function along the Pareto front for all three side-vent-channel architectures. Furthermore, it is seen that the replacement of the all-steel side-vent-channels with their sandwich-structure counterparts improves the overall blast-mitigation performance of this solution. As far as the relative performance of the linear and bi-linear side-vent-channel architectures is concerned, the bi-linear architecture appears to be slightly superior.

Optimal Side-vent-channel Sandwich-structure Wall Architectures along the Pareto Front: Variation of the four design variables, for the cases of linear and bi-linear variation in the aluminum-foam density through the sandwich-structure core, along the Pareto front is depicted in Figures 16(a)–(d). Examination of the results displayed in these figures reveals that:

(a) The steel face-sheet thickness, $DV1$, varies only slightly (by 2–2.5%) along the Pareto front for both the linear and bi-linear architectures of the sandwich-structure core. Furthermore, slightly thinner steel face-sheet thicknesses are preferred in the momentum percent reduction

Fig. 15. Variations of the two components of the objective function along the Pareto front. The three Pareto fronts correspond to the cases of the following side-vent-channel architectures: all-steel, sandwich-structure with linear density variation, and sandwich-structure with bi-linear density variation.
dominated portion of the Pareto front. Slightly higher (ca. 1.5%) values for \( DV1 \) are obtained in the bi-linear case over the entire Pareto front;

(b) Variation of the aluminum foam-core thickness, \( DV2 \), is quite significant (by 8–9%) along the Pareto front for both the linear and bi-linear architectures of the sandwich-structure core. In addition, \( DV2 \) acquires the highest values in the momentum percent reduction dominated portion of the Pareto front. Moderately lower (ca. 4%) values for foam-core thickness are obtained in the bi-linear case over the entire Pareto front;

![Graphs showing variations of four design variables along the Pareto front](image)

Fig. 16. Variations of the four design variables along the Pareto front: (a) steel face-sheet thickness, \( DV1 \); (b) aluminum foam-core thickness, \( DV2 \); (c) foam average density, \( DV3 \); and (d) (constant) rate of change of foam density starting at the inward-facing side of the sandwich-structure core, \( DV4 \).

(c) The aluminum-foam core mean density, \( DV3 \), varies modestly (by 5–6%) along the Pareto front for both the linear and bi-linear architectures of the sandwich-structure core and acquires the lowest values in the momentum percent reduction dominated portion of the Pareto
front. Furthermore, slightly higher (ca. 2.5%) values for $DV3$ are obtained in the bi-linear case over the entire Pareto front; and

(d) The rate of change of the foam density with core thickness at the inward-facing side, $DV4$, takes on negative values along the entire Pareto front for both the linear and bi-linear architectures of the sandwich-structure core. In other words, the largest aluminum-foam density levels are encountered at the inward-facing side of the core. Furthermore, as expected, the density gradient for the bi-linear architecture has a magnitude which is nearly twice as large as that for the linear architecture. Variations in $DV4$ along the Pareto front are quite significant (ca. 10%) and their magnitude is largest in the acceleration percent reduction dominated portion of the Pareto front.

Optimized aluminum-foam core linear density profiles in the acceleration percent reduction dominated and momentum percent reduction dominated ends of the Pareto front are displayed in Figures 17(a)-(b). The corresponding bi-linear density profiles are depicted in Figures 17(c)-(d). Examination of the results displayed in Figures 17(a)-(d) reveals that: (a) in the case of the linear foam-density profile, the foam density decreases with distance from the inward-facing side of the sandwich-structure core and is consistently higher for the acceleration percent reduction dominated design, Figures 17(a)-(b); (b) the same observation regarding the consistently higher aluminum-foam core density being found in the acceleration percent reduction dominated design can be made in the case of bi-linear foam-density profile, Figures 17(c)-(d); and (c) consistently higher foam-density values are encountered in the bi-linear density variation case, particularly in the lowest-density regions of the sandwich-structure core.

Optimal-designs Statistical Variation: For the most part, the present computational investigation was of a deterministic character. However, due to the granular nature of soil and the statistical aspects of the soil-particle size/shape and spatial location, computational analyses conducted under nominally identical conditions of the soil density yielded different results. Differences in these results are used in the present section to assess the extent of statistical variability of the optimal SBS side-vent-channel designs and their performance, as discussed in the previous section. Specifically, the designs corresponding to points $A$ and $B$, for the cases of linear and balanced bi-linear variation in the aluminum-foam core density, in Figure 15 are investigated in this section. By carrying out a standard statistical analysis involving the use of the maximum likelihood estimation method, it was determined that the statistical variability of both optimal designs, for both types of aluminum-foam density variation, can be represented using a bi-variate normal distribution function. The results of this analysis are shown, as contour plots, in Figures 18(a)-(d). The results displayed in Figures 18(a)-(b) pertain, respectively, to the optimal designs $A$ and $B$ in the case of linear aluminum-foam density variation through the sandwich-structure core. The corresponding results for the case of balanced bi-linear variation of the aluminum-foam density are depicted in Figures 18(c)-(d). Examination of the results displayed in these figures and their subsequent analysis revealed that:

(a) In the aluminum-foam linear density variation case, for the optimal design $A$ (i.e. for the design which maximizes the percent momentum reduction), Figure 18(a), the two components of the standard deviation (one associated with the momentum reduction and the other with the acceleration reduction) take on the values of 0.45 and 0.41. This finding suggests that, with a
statistical probability of 99.97%, the optimal design $A$ offers percent momentum reduction in a 3.82–6.52 range, and percent acceleration reduction in a 2.41–4.87 range;

(b) In the aluminum-foam linear density variation case, for the optimal design $B$ (i.e. for the design which maximizes the percent acceleration reduction), Figure 18(b), the two components of the standard deviation take on the values of 0.44 and 0.40. This finding suggests that, with a statistical probability of 99.97%, the optimal design $B$ offers percent momentum reduction in a 3.10–5.74 range, and percent acceleration reduction in a 3.77–6.17 range;

![Graphs showing optimized aluminum-foam core linear density profiles](image)

**Fig. 17.** Optimized aluminum-foam core linear density profiles in the: (a) momentum percent reduction dominated; and (b) acceleration percent reduction dominated ends of the Pareto front; (c) and (d) the corresponding bi-linear density profiles.

(c) In the aluminum-foam bi-linear density variation case, for the optimal design $A$, Figure 18(c), the two components of the standard deviation take on the values of 0.44 and 0.41. This finding suggests that, with a statistical probability of 99.97%, the optimal design $A$ offers percent
momentum reduction in a 3.86–6.50 range, and percent acceleration reduction in a 2.41–4.87 range; and

(d) In the aluminum-foam bi-linear density variation case, for the optimal design $B$, Figure 18(d), the two components of the standard deviation take on the values of 0.44 and 0.40. This finding suggests that, with a statistical probability of 99.97%, the optimal design $B$ offers percent momentum reduction in a 3.10–5.74 range, and percent acceleration reduction in a 3.77–6.17 range.

![Fig. 18. Bi-variate normal-distribution probability density function in the case of a linear foam-density variation through the sandwich-structure core for: (a) the optimal side-vent-channel design $A$, i.e., the design which maximizes the percent momentum reduction; and (b) the optimal side-vent-channel design $B$, i.e., the design which maximizes the percent acceleration reduction; (c) and (d) the corresponding results in the case of a bi-linear variation of the foam density through the sandwich-structure core.](image-url)
4. Summary and Conclusions

Based on the results obtained in the present work, the following main summary remarks and conclusions can be drawn:

1. Our recently proposed side-vent-channel concept/solution for mitigation of the blast-loads resulting from a shallow-buried mine detonated underneath a light tactical vehicle has been extended to include modifications in the side-vent-channel wall architecture. Specifically, instead of using all-steel side-vent-channels, side-vent-channels made of sandwich structures consisting of steel face sheets and aluminum foam cores are considered.

2. To assess the potential of the modified side-vent-channel solution, a comprehensive finite-element/discrete-particle/continuum-fluid computational analysis is carried out for a problem involving detonation of a shallow-buried landmine and the interaction of the detonation products and soil ejecta with the target structure.

3. To maximize the blast-mitigation effect of the modified side-vent-channel design, as quantified by reduction of: (i) the momentum transferred to; and (ii) the maximum acceleration acquired by the targeted vehicle, a bi-objective engineering-optimization analysis of the side-vent-channel wall core architecture was carried out.

4. The results obtained show that replacement of all-steel side-vent-channels with their sandwich-structure counterparts has a positive effect on the blast-mitigation potential of the proposed solution. In addition, the results obtained revealed that grading of the aluminum-foam density through the core thickness can provide additional increment in the side-vent-channel blast-mitigation potential.

5. To deal with trade-off between the two conflicting objectives, a set of optimal Pareto designs was identified. In these designs, a given level of the blast-mitigation potential with respect to one objective is attained at the maximum level of the blast-mitigation potential with respect to the other objective. This approach revealed, quantitatively, the extent of trade-off which must be accepted when trying to optimize the side-vent-channel design with respect to its maximum potential for reduction of the momentum transferred to, and the acceleration acquired by, the target vehicle.

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