The Probability of Non-ruin of an Insurance Company with Advertising Expenses by MHull Insurance of 10 Top Insurance companies of Ukraine. I.

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Abstract

A main result of the paper is constructing an equation that allows calculating the probability of non-ruin for the classical model of risk when an insurance company has promotional costs. Some numerical results for the probability of non-ruin by MHull of 10 Top Insurance companies of Ukraine for the period 01/01/2015 to 06/30/2015 based on the developed formulae are provided. Some conclusions about the influence of advertising company on the probability of non-ruin were carried out. The first part of the paper analyses the model without investments in financial (B,S)-market, the second one will deal with investments on this market.

Keywords: Probability of non-ruin; Advertising expenses; Classical risk model; Poisson process

1. Introduction

The attraction of insurants by the devices of different advertising forms is an important tool of competitive struggle in the insurance market. Using mathematical models within the estimation of insurance company's activity would provide managers with a formulation of the administrative decisions adapted to the environment perturbations. Last years a significant interest emerged in the analyses of an insurance company activity with a promotional campaign. Akhmedova and Terpugov (2001) proposed the insurance company model where the advertising costs were proportional to the capital, and the efficiency conditions of advertising were founded. Akhmedova et al (2002) studied the case when the part of the capital on advertising depend on the time, under these conditions the optimal solution to find the maximum capital at the end of the reporting period was founded. Zhmykhova (2011) resolved the optimal solution for finding the optimal part of the funds which the insurance company devotes to advertising in the case when the company operates

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in the financial (B, S)-market. Katz (2007) analyzed the impact of advertising consumed on average equity of the company for the classical risk model, the optimization problem of this work has been reduced to the finding the switching points for management of advertising. The main difference between the mentioned conditions and obtained earlier is that in this work the problem of a finding of non-ruin probability was solved that helps analysts to make the correct conclusion for insurance company investment and tariff policy. Moreover, in the work the connection order $\frac{1}{2}$ is used bearing in mind the fact that it is more expedient to consider the return on advertising activities, in our case the receipt of premiums from customers, rather than the value of the insurance company capital but of the market size except of the company (Sethi et al, 2008). The work uses previous research of authors (Bondarev and Boldyreva, (2012)) with some changes and improves results with using numerical data. Our results are illustrated by an example of M Hull insurance. Motor Hull Insurance provides protection against risks, leading to partial damage or total loss of the vehicle (Unicredit bank website).

On the basis of the recorded facts and the resulting model, in this paper the work of 10 leading insurance companies in Ukraine during the period from 01.01.2015 on 06.30.2015 is analyzed and the effect of deductions for advertising on the non-ruin probability of insurance companies is studied.

2. Classical Model of Risk with Advertising Expenses

2.1 Deriving an equation for the insurance company non-bankruptcy probability with the deductions on advertising on an infinite time interval $t \in [0, \infty)$:

We start this section by introducing some basic terms.

**Definition 2.1** The model is called Cramer-Lundberg model or classical risk model when the surplus is modeled by the relation

$$X_t = x + ct - \sum_{k=1}^{N_t} Y_k,$$

where $x(x \geq 0)$ is the initial capital, $c > 0$ is the premium rate, $N_t$ is a Poisson process with rate $\lambda(\lambda > 0)$ (Rolski et al. (1999)) which describes the number of claims in $(0, t]$ and $(Y_k : k \in \mathbb{N})$ is a sequence of positive random variables independent of $N_t$ which describes the claim sizes. For simplicity, we let $Y = Y_1$ be a generic random variable and we denote the distribution function of $Y$ by $F(y)$.

**Definition 2.2** The time of ruin

$$\tau = \tau(x) = \inf \{t \geq 0 : X_t < 0\}$$

is the first time where the surplus becomes negative.

Let the insurance company starts advertising activities to attract new customers, some part of the income, exactly $\delta > 0 (0 < \delta < 1)$, is deducted on advertising. The intensity of premium income will
increase by \( j(t) \), at the same time the intensity of claims to the insurance company increases by the \( j(t) \). Then the capital of the company is described by the equation

\[
\xi_x(t+\Delta t) = \xi_x(t) + (1-\delta) \left[ c(1+j(t))\Delta t - \int_0^t \alpha v_{\lambda(t+\delta)}(d\alpha, ds) \right].
\]

(1)

here \( \xi_x(0)=x \). We have also used the representation of compound Poisson process as stochastic integral by Poisson measure (Bondarev, 2002).

On the other side, equation (1) can be rewritten as

\[
d\xi_x(t) = (1-\delta) \left[ c(1+j(t))dt - \int_0^t \alpha v_{\lambda(t+\delta)}(d\alpha, dt) \right].
\]

(2)

**Theorem 2.1** If the evolution of the insurance company capital is described by equation (2), then the insurance company non-bankruptcy probability with the deductions on advertising is a solution of the integro-differential equation on an infinite time interval \( t \in [0, \infty) \):

\[
\frac{(1-\delta)c(1+j(t))}{\lambda(1+j(t))} \phi(x) = \phi(x) - \int_0^x \phi(x-y) dF(y)
\]

(3)

with the limit condition \( \lim_{x \to \infty} \phi(x) = 1 \).

**Proof:** To prove this theorem we have used the methods represented in the book by Leonenko et al. (1995). So, for the first jump of the process

\[
dR(t) = \int_0^t \alpha v_{\lambda(t+\delta)}(d\alpha, dt)
\]

the capital of the company is described by the equation

\[
d\xi_x(t) = (1-\delta) c(1+j(t))dt,
\]

from which we derive

\[
\xi_x(t) = (1-\delta) c(1+j(t))t + x.
\]

Let the first jump of the process \( R(t) \) is occurring at time \( \tau = s \), and its value is equal to \( y \). Until the time \( \tau \) the ruin will not happen. In order to ruin of company will not happen from the time point \( \tau \), it is necessary and sufficient that the value of the jump does not exceed the value of the existing capital, so we will demand the fulfillment of the condition

\[
y \leq x + (1-\delta) c(1+j(t))t.
\]

The time point of the first jump has an exponential distribution with the parameter \( \lambda(1+j(t)) \), by virtue of the fact that process \( N(t) \) is Poisson with the parameter \( \lambda(1+j(t)) \). Then we apply the formula of total probability and derive

\[
\phi(x) = \int_0^\infty \lambda(1+j(t)) e^{-\lambda(1+j(t))t} x (1-\delta)(1+j(t)) t \phi(x+ (1-\delta) c(1+j(t))s-y) dF(y) ds.
\]

(4)

If we make the change of variable in integral (4), we obtain

\[
\phi(x) = \frac{\lambda(1+j(t))}{(1-\delta)c(1+j(t))} \int_x e^{-\lambda(1+j(t))u} \phi(u-y) dF(y) du,
\]

(5)

Differentiating equation (5) with respect to \( x \), we get
\[ \phi(x) = \left[ \frac{\lambda(1+j(\delta))}{(1-\delta)c(1+j(\delta))} \right]^2 e^{\frac{\lambda(1+j(\delta))x}{[1-\delta](1+j(\delta))}} \int_{x}^{e^{\frac{\lambda(1+j(\delta))u}{[1-\delta](1+j(\delta))}}} \phi(u-y) dF(y) du - \frac{\lambda(1+j(\delta))}{(1-\delta)c(1+j(\delta))} \int_{0}^{x} \phi(x-y) dF(y). \] 

(6)

By substituting (5) to (6) we have obtained the statement of the theorem.

2.2 Analytical solution of the integro-differential equation (3) in the case of exponentially distributed claims

Let claims to insurance company have exponential distribution with parameter \( a \) \((a>0)\), so \( F(y) = 1-e^{-ay} \). We rewrite the equation (3) as

\[
(1-\delta)c(1+j(\delta))\phi(x) = \lambda(1+j(\delta))\phi(x) - \lambda(1+j(\delta)) \int_{0}^{x} \phi(x-y) a e^{-ay} dy.
\]

(7)

Differentiating the equation (7) with respect to \( x \), we get

\[
(1-\delta)c(1+j(\delta))\phi'(x) = \lambda(1+j(\delta))\phi(x) - \lambda(1+j(\delta))\phi(0) a e^{-ax} + a \lambda(1+j(\delta)) \int_{0}^{x} \phi(x-y) a e^{-ay} dy + \phi(0) a e^{-ax} - \phi(x)a.
\]

(8)

Making the substitution from (7) to (8), we get

\[
(1-\delta)c(1+j(\delta))\phi'(x) = \lambda(1+j(\delta))\phi(x) - a(1-\delta)c(1+j(\delta))\phi(x).
\]

(9)

A solution of the equation (9) is

\[
\phi(x) = Ae^{\left( -\frac{\lambda(1+j(\delta))}{(1-\delta)c(1+j(\delta))} \right) x} + B,
\]

(10)

where constants \( A \) and \( B \) can be determined with the usage of boundary conditions:

\[
\lim_{x \to \infty} \phi(x) = 1,
\]

\[
(1-\delta)c(1+j(\delta))\phi'(0) = \lambda(1+j(\delta))\phi(0).
\]

So, the analytical solution of (10) is

\[
\phi(x) = 1 - \frac{\lambda(1+j(\delta))}{a(1-\delta)c(1+j(\delta))} e^{\left( -\frac{\lambda(1+j(\delta))}{(1-\delta)c(1+j(\delta))} \right) x}.
\]

(11)

Besides, in virtue of a clear profit condition we demand the fulfillment of the condition

\[
\frac{\lambda(1+j(\delta))}{(1-\delta)c(1+j(\delta))} - a < 0,
\]

(12)

whereof, we have the restriction to \( \delta > 0 \)

\[
\delta < 1 - \frac{\lambda(1+j(\delta))}{ac(1+j(\delta))},
\]

3. Numerical Results by Hull Insurance of 10 Top Ukrainian Insurance companies
We present numerical results by MHull of 10 Top Ukrainian Insurance companies used the developed formulae (11) for the probability of non-ruin. Motor Hull Insurance provides protection against risks, which lead to partial damage or total loss of the vehicle. Necessary data for analysis are presented in the following table.

**Table 1** The probability of non-ruin of 10 Top Ukrainian Insurance companies by MHull

<table>
<thead>
<tr>
<th>Company</th>
<th>Exp(a)</th>
<th>Premiums, c</th>
<th>Intensity of claims, λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oranta</td>
<td>Exp(0,0000917)</td>
<td>44337,78</td>
<td>1,944444</td>
</tr>
<tr>
<td>VUSO</td>
<td>Exp(0,0000848)</td>
<td>132218,3</td>
<td>4,872222</td>
</tr>
<tr>
<td>Illichevske</td>
<td>Exp(0,0000824)</td>
<td>58487,22</td>
<td>2,338889</td>
</tr>
<tr>
<td>Providna</td>
<td>Exp(0,0000748)</td>
<td>256393,3</td>
<td>10,51667</td>
</tr>
<tr>
<td>Arsenal - insurance</td>
<td>Exp(0,0000742)</td>
<td>459008,3</td>
<td>18,47778</td>
</tr>
<tr>
<td>UPSK</td>
<td>Exp(0,0000721)</td>
<td>113874,4</td>
<td>3,294444</td>
</tr>
<tr>
<td>Alfa-insurance</td>
<td>Exp(0,0000718)</td>
<td>170552,2</td>
<td>6,744444</td>
</tr>
<tr>
<td>Persha</td>
<td>Exp(0,0000688)</td>
<td>36177,78</td>
<td>1,13889</td>
</tr>
<tr>
<td>Aha-insurance</td>
<td>Exp(0,0000684)</td>
<td>1743217</td>
<td>62,09444</td>
</tr>
<tr>
<td>PZU-Ukraine</td>
<td>Exp(0,0000683)</td>
<td>442853,3</td>
<td>14,37778</td>
</tr>
</tbody>
</table>

The level of initial capital \( x = 1 \) 500 000

The period of modeling 1/2 of year

Let us turn to the study of the available data. We constructed graphs of the non-ruin probability as a function of the initial capital with fixed proportion of funds for advertising for each company, as well as graphs of the insurance companies non-ruin probability of the deposited share of advertising at a fixed value of the initial capital. In accordance with regulatory requirements for the implementation of insurance activity in Ukraine the minimum authorized capital of the insurance company, engaged in types of insurance other than life insurance, is established in the amount of 1 million €, and for the insurance company, engaged in life insurance, it is equivalent 1.5 million € in UAH. The value of invested money in advertising should not exceed 1%. We consider the special case of advertising impact on the activities of companies, namely when \( j(\delta) = j_1(\delta) = \sqrt{\delta} \). (Sethi et al, 2008)

Let us consider the first company in the list – Oranta and fix \( \delta = 0,01 \). We can note with the help of Figure 1, that the amount of capital of 1.5 million UAH more than enough to ensure that the company will continue to operate successfully (1 million EURO would be enough for the more). Moreover, for the operation of the company would be enough 200 thousand UAH, which confirms the Figure 2.
Now fix the initial capital level at 1.5 million UAH. Considering the dependence of the non-ruin probability of the deposited value of advertising (Fig.3), we note that advertising will likely be used here for attracting of the number of new customers and not for strengthening the existing well-being. Also, the graph clearly shows that when it reaches a fairly high level, close to 50%, advertising will help to reduce the likelihood of non-bankruptcy and thus impair its financial condition.

Furthermore, to study the behavior of the non-ruin probability the authors considered it as a function that depends both on the initial capital and advertising costs. In order to understand better the behavior of the resulting surface, the initial capital was limited to the value of 500 thousand UAH. The resulting surface is shown in Figure 4. The graph shows that the advertising costs will only reduce its probability of non-ruin in classical risk model.

We found a fairly high similarity in the obtained results for the other companies of the top 10, so we are not listed all of them in this article. When we fixed the initial capital level at 1.5 million UAH and $\delta=1\%$, we have found that for each of the companies for this selected risk model, the probability of non-ruin is equal to 1. All of these companies have a high degree of reliability.
Fig.3. The probability of non-bankruptcy of Oranta, $\delta \in (0,0.5)$

Fig.4. The probability of non-bankruptcy as function of initial capital and costs on advertising

4. Conclusions

The article examines activity of insurance companies in terms of the classical risk model with the cost of advertising. For this model we derive an integro-differential equation for the insurance company non-ruin probability. The analytic solution of the equation is found in the case where the sizes of insurance claims are described by an exponential distribution. The results have been applied in the analysis of the activities by MHull of 10 Top Insurance companies of Ukraine for the period 01/01/2015 to 06/30/2015.
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Unicredit bank website