Uniform Convergence of Wavelet-based Expansions of Random Processes

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Abstract

We consider a wavelet-based expansion of a second-order stochastic process in a wavelet-based random series with independent terms. Conditions for uniform convergence with probability 1 of such expansion for stochastic processes from a wide class which includes stationary and non-stationary processes are obtained. This result is a counterpart of a theorem from Kozachenko and Turchin (2009), where only stationary processes were considered.

Keywords: Wavelets; Random functional series; Expansion

1. Introduction

Wavelet-based expansions constitute an interesting class of representations of random processes. Wavelet-based expansions with uncorrelated or independent terms are especially remarkable and useful since they are convenient for simulation and approximation of random processes and possess some other special properties.

Various wavelet-based expansions of stochastic processes were studied by Walter and Zhang (1994), Meyer et al. (1999), Pipiras (2004), Zhao et al. (2004). Kozachenko and Turchyn (2008) proved a theorem about a wavelet-based series expansion for a wide class of second-order stochastic process. Namely, a centered second-order process $X(t)$ with the correlation function

$$R(t, s) = \int_{\mathbb{R}} u(t, y)u(s, y)dy$$

($u(t, \cdot) \in L_2(\mathbb{R})$) can be represented as the series

$$X(t) = \sum_{k \in \mathbb{Z}} \xi_k a_k(t) + \sum_{j=0}^{\infty} \sum_{k \in \mathbb{Z}} \eta_{jk} b_{jk}(t)$$

which converges in $L_2(\Omega)$ for any fixed $t$, where

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\[ a_{0k}(t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} u(t,y) \hat{\phi}_{0k}(y) dy, \quad b_{jk}(t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} u(t,y) \hat{\psi}_{jk}(y) dy, \]

\( \phi(y) \) is a \( f \)-wavelet, \( \psi(y) \) is the corresponding \( m \)-wavelet, \( \hat{\phi}_{0k}(y) \) and \( \hat{\psi}_{jk}(y) \) are the Fourier transforms of \( \phi_{0k}(y) = \phi(y - k) \) and \( \psi_{jk}(y) = 2^{j/2} \psi(2^j y - k) \) correspondingly, \( \xi_{0k} \) and \( \eta_{jk} \) are uncorrelated random variables.

Properties of this expansion and its generalizations were studied by Kozachenko and Turchyn (2008), Kozachenko et al. (2011), Turchyn (2011), Turchyn (2016). In the paper by Kozachenko and Turchyn (2009) uniform convergence of expansion (1) was studied. Conditions for uniform convergence with probability 1 were found in the case where terms of series (1) are independent and the process is stationary. Conditions for uniform convergence in probability of series (1) were also found.

In this paper we obtain a counterpart of the result from Kozachenko and Turchyn (2009) about uniform convergence with probability 1 of expansion (1) for a class of stochastic processes which includes wide families of stationary and non-stationary processes.

### 2. Expansion of a Random Process in a Series with Uncorrelated Terms

Let \( \phi = \{ \phi(x), x \in \mathbb{R} \} \) be a function from \( L_2(\mathbb{R}) \), and let \( \hat{\phi}(y) \) be the Fourier transform of \( \phi \):

\[ \hat{\phi}(y) = \int_{\mathbb{R}} e^{-i y x} \phi(x) dx. \]  \hspace{1cm} (2)

A function \( \phi \) is called a \( f \)-wavelet if the following conditions hold:

1) the system of functions \( \{ \phi(x - k), k \in \mathbb{Z} \} \) is orthonormal in \( L_2(\mathbb{R}) \);

2) there exists a \( 2\pi \)-periodic function \( m_0 \in L_2([0,2\pi]) \) such that

\[ \hat{\phi}(2y) = m_0(0) \hat{\phi}(y); \]  \hspace{1cm} (3)

3) \( \hat{\phi}(0) \neq 0 \) and \( \hat{\phi}(y) \) is continuous at zero.

Define a function \( \hat{\psi}(y) \):

\[ \hat{\psi}(y) = m_0 \left( \frac{y}{2} + \pi \right) \exp\{-iy/2\} \hat{\phi}\left(\frac{y}{2}\right). \]

Let \( \psi(x) \) be the inverse Fourier transform of the function \( \hat{\psi}(y) \), i.e.

\[ \psi(x) = \frac{1}{2\pi} \int_{\mathbb{R}} e^{iyx} \hat{\psi}(y) dy. \]

The function \( \psi(x) \) is called a \( m \)-wavelet which corresponds to the \( f \)-wavelet \( \phi \).

Denote

\[ \phi_{jk}(x) = 2^{j/2} \phi(2^j x - k), \quad \psi_{jk}(x) = 2^{j/2} \psi(2^j x - k), \quad j \in \mathbb{Z}, \quad k \in \mathbb{Z}. \]

It is known that the system of functions \( \{ \phi_{0k}, \psi_{jk}, j = 0, \infty, k \in \mathbb{Z} \} \) is an orthonormal basis in \( L_2(\mathbb{R}) \).

Obviously the system \( \{ \phi_{0k}/\sqrt{2\pi}, \psi_{jk}/\sqrt{2\pi}, j = 0, 1, \ldots; k \in \mathbb{Z} \} \) also is an orthonormal basis in \( L_2(\mathbb{R}) \).

We need two following propositions which are proved in the paper by Kozachenko and Turchyn (2008).

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Theorem 2.1. Suppose that \( X = \{X(t), t \in \mathbb{R}\} \) is a centered stochastic process, \( E|X(t)|^2 < \infty, \; t \in \mathbb{R} \), \( R(t,s) = EX(t)X(s) \), \( \phi = \{\phi(x), x \in \mathbb{R}\} \) is a \( f \)-wavelet, \( \psi = \{\psi(x), x \in \mathbb{R}\} \) is the corresponding \( m \)-wavelet, 
\[
\phi_{jk}(x) = 2^{j/2}\phi(2^j x - k), \quad \psi_{jk}(x) = 2^{j/2}\psi(2^j x - k), \quad j \in \mathbb{Z}, \; k \in \mathbb{Z}.
\]
If \( R(t,s) \) can be represented as 
\[
R(t,s) = \int_{\mathbb{R}} u(t,y)\overline{u(s,y)}dy,
\]
where \( \int_{\mathbb{R}} |u(t,y)|^2 dy < \infty \) for all \( t \in \mathbb{R} \), then the process \( X(t) \) can be represented in the form of the series 
\[
X(t) = \sum_{k \in \mathbb{Z}} \xi_{0k}a_{0k}(t) + \sum_{j=0}^{\infty} \sum_{k \in \mathbb{Z}} \eta_{jk}b_{jk}(t), \tag{4}
\]
where the series converges in \( L_2(\Omega) \) for all \( t \in \mathbb{R} \), 
\[
a_{0k}(t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} u(t,y) \overline{\phi_{0k}(y)} dy = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} u(t,y) \phi(y) e^{iky} dy, \tag{5}
\]
\[
b_{jk}(t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} u(t,y) \overline{\psi_{jk}(y)} dy = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} u(t,y) 2^{-j/2} \exp\{i \frac{y}{2^j} k\} \psi(y/2^j) dy, \tag{6}
\]
\( \hat{\phi}_{0k}(y) \) and \( \hat{\psi}_{jk}(y) \) are the Fourier transforms of \( \phi_{0k}(x) \) and \( \psi_{jk}(x) \) correspondingly, \( \xi_{0k}, \eta_{jk} \) are centered random variables such that 
\[
E\xi_{0k} = \delta_{kl}, \quad E\eta_{nk} = \delta_{mn}\delta_{kl}, \quad E\xi_{0k}\eta_{nl} = 0, \quad \delta_{kl} \text{ is the Kronecker symbol.}
\]

Corollary 2.1. Suppose that a centered second-order stationary process \( X = \{X(t), t \in \mathbb{R}\} \) has the spectral density \( f(y), \; \phi = \{\phi(x), x \in \mathbb{R}\} \) is a \( f \)-wavelet, \( \psi = \{\psi(x), x \in \mathbb{R}\} \) is the corresponding \( m \)-wavelet. Then \( X(t) \) can be represented as a mean square convergent series 
\[
X(t) = \sum_{k \in \mathbb{Z}} \xi_{0k}a_{0k}(t) + \sum_{j=0}^{\infty} \sum_{k \in \mathbb{Z}} \eta_{jk}b_{jk}(t), \tag{7}
\]
and the coefficients \( a_{0k}(t) \), \( b_{jk}(t), k \in \mathbb{Z}, \; j = 0,1, ..., \) can be represented as 
\[
a_{0k}(t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} g(y) \exp\{-iy(t - k)\} \phi(y) dy, \tag{8}
\]
\[
b_{jk}(t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} g(y) \exp\{-iy\left(t - \frac{k}{2^j}\right)\} \psi(y/2^j) dy, \tag{9}
\]
where \( g(y) = \sqrt{f(y)} \), random variables \( \xi_{0k}, \eta_{jk} \) from (7) are such that 
\[
E\xi_{0k} = E\eta_{jk} = 0, \quad E\xi_{0k}\xi_{0l} = \delta_{kl}, \quad E\eta_{jk}\eta_{jm} = \delta_{jl}\delta_{km}, \quad E\xi_{0k}\eta_{nl} = 0.
\]

3. Conditions of Uniform Convergence with Probability 1

Suppose that random variables \( \xi_{0k}, \eta_{jk} \) in expansion (4) are independent. We will impose certain restrictions on the process \( X(t) \) and the wavelet which guarantee that for \( h \) small enough
\[ \sup_{|t-s| \leq h, \ t, s \in T} |a_{0k}(t) - a_{0k}(s)| \leq a^*_0k \sigma(h) \]  
and  
\[ \sup_{|t-s| \leq h, \ t, s \in T} |b_{jk}(t) - b_{jk}(s)| \leq b^*_jk \sigma(h) , \]

where \( \sigma(h) \) is a "good" function and  
\[ \sum_{k \in \mathbb{Z}} (a^*_0k)^2 + \sum_{j=0}^{\infty} \sum_{k \in \mathbb{Z}} (b^*_jk)^2 < \infty . \]

In this paper we will show that under these conditions series (4) converges uniformly with probability 1 on any bounded segment \( T \).

**Remark.** Conditions (10) and (11) imply  
\[ \sup_{|t-s| \leq h, \ t, s \in T} (E|X(t) - X(s)|^2)^{1/2} \leq C \sigma(h) \]
(where \( C \) is a constant). The case where  
\[ \sigma(h) = \frac{1}{(\ln(\varepsilon a + 1/h))^{a/2}} \]
(\( \varepsilon > 0 \)) was considered in the paper by Kozachenko and Turchin (2009). In this paper we will consider only the case \( \sigma(h) = h \).

**Definition 3.1.** We say that a continuous function \( \sigma = \{\sigma(h), h > 0\} \) satisfies condition A if \( \sigma(h) \) increases for \( h > 0 \), \( \sigma(+0) = 0 \) and for \( \varepsilon > 0 \) small enough  
\[ \int_0^\varepsilon |\ln \sigma^{-1}(v)|^{1/2} \, dv < \infty \]
(\( \sigma^{-1}(v) \) is the inverse function for \( \sigma(v) \)).

**Remark.** Let us note that the function \( \sigma(h) = h \) \( (h > 0) \) satisfies condition A.

**Theorem 3.1.** (Kozachenko and Turchin (2009)) Suppose that a process \( X = \{X(t), \ t \in \mathbb{R}\} \) satisfies conditions of Theorem 2.1 together with a \( f \)-wavelet \( \phi(y) \) and the corresponding \( m \)-wavelet \( \psi(y) \), random variables \( \xi_{0k}, \eta_{jk}, j = 0, \infty, k \in \mathbb{Z} \) in (4) are independent and a function \( \sigma = \{\sigma(h), h > 0\} \) satisfies condition A. If for \( h \) small enough  
\[ \sup_{|t-s| \leq h, \ t, s \in T} |a_{0k}(t) - a_{0k}(s)| \leq a^*_0k \sigma(h) , \]
\[ \sup_{|t-s| \leq h, \ t, s \in T} |b_{jk}(t) - b_{jk}(s)| \leq b^*_jk \sigma(h) , \]
where  
\[ \sum_{k \in \mathbb{Z}} (a^*_0k)^2 + \sum_{j=0}^{\infty} \sum_{k \in \mathbb{Z}} (b^*_jk)^2 < \infty , \]
then series (4) converges uniformly with respect to \( t \in T \) with probability 1 on any bounded closed interval \( T \subset \mathbb{R} \).

**Remark.** Random variables \( \xi_{0k}, \eta_{jk}, j = 0, \infty, k \in \mathbb{Z} \) in (4) are independent, for instance, in the case where \( X(t) \) is a Gaussian stochastic process.
Lemma 3.1. Let $X = \{X(t), t \in \mathbb{R}\}$ be a stochastic process which satisfies conditions of Theorem 2.1 together with a $f$-wavelet $\phi(y)$ and the corresponding $m$-wavelet $\psi(y), T > 0$. Suppose that for any fixed $t \in [0,T]$ the function $u(t,y)$ is absolutely continuous with respect to $y$ and

$$\sup_{y \in \mathbb{R}} |u(t,y)| < \infty,$$

$\hat{\phi}(y), \hat{\psi}(y)$ are absolutely continuous and bounded on $\mathbb{R}$, there exist derivatives $u'_y(t,y), \hat{\psi}(y), \phi'(y)$ for all $y \in \mathbb{R}, t \in [0,T]$ and

$$\sup_{y \in \mathbb{R}} |\hat{\psi}'(y)| \leq M < \infty,$$

where

$$u(t,y)|\hat{\phi}'(y)| \to 0 \text{ as } |y| \to \infty, \quad u(t,y)|\hat{\psi}(y/2^j)| \to 0 \text{ as } |y| \to \infty, \quad j = 0, 1, \ldots, \quad t \in [0,T],$$

$$|u(t_1,y) - u(t_2,y)| \leq v(y)|t_1 - t_2|, \quad t_1, t_2 \in [0,T],$$

$$|u'_y(t_1,y) - u'_y(t_2,y)| \leq w(y)|t_1 - t_2|, \quad t_1, t_2 \in [0,T],$$

Then the following inequalities hold true:

$$|a_{00}(t_1) - a_{00}(t_2)| \leq C_{a,0} |t_1 - t_2|,$$

$$|a_{0k}(t_1) - a_{0k}(t_2)| \leq \frac{C_{a,1}}{|k|} |t_1 - t_2|, \quad k \neq 0,$$

$$|b_{j0}(t_1) - b_{j0}(t_2)| \leq \frac{C_{b,0}}{2^{3j/2}} |t_1 - t_2|,$$

$$|b_{jk}(t_1) - b_{jk}(t_2)| \leq \frac{C_{b,1}}{2^{j/2}|k|} |t_1 - t_2|, \quad k \neq 0,$$

$t_1, t_2 \in [0,T]$, where $a_{0k}(t), b_{jk}(t)$ are defined by (5) and (6), $C_{a,0}, C_{a,1}, C_{b,0}, C_{b,1}$ are some constants.

Proof. Let us prove (19). Let $k \neq 0$. Integrating by parts and using (15) we obtain:

$$|b_{jk}(t_1) - b_{jk}(t_2)| = \frac{1}{\sqrt{2\pi2^{j/2}}} \int_{\mathbb{R}} (u(t_1,y) - u(t_2,y)) \overline{\psi(y/2^j)} d\left(\exp\left(iyk/2^j\right)\right)$$

$$\leq \frac{2^{j/2}}{\sqrt{2\pi} |k|} \int_{\mathbb{R}} \exp\left(iyk/2^j\right) |(u'_y(t_1,y) - u'_y(t_2,y)) \overline{\psi(y/2^j)}$$

$$+ (u(t_1,y) - u(t_2,y)) \overline{\psi(y/2^j)/2^j} |dy|$$
\[ \leq \frac{2^{j/2}}{\sqrt{2\pi}} | \int_{\mathbb{R}} \left( u'_y(t_1, y) - u'_y(t_2, y) \right) \hat{\psi}(y/2^j) \, dy \]
\[ + \int_{\mathbb{R}} \left( u(t_1, y) - u(t_2, y) \right) \hat{\psi}(y/2^j) \, dy \].

Since \(|\hat{\psi}(y/2^j)| = |\hat{\psi}(y/2^j) - \hat{\psi}(0)| \leq M|y|/2^j\) now (19) easily follows. Inequalities (16)–(18) can be obtained in a similar way.

The following statement is a particular case of Lemma 3.1.

**Lemma 3.2.** Suppose that \( X = \{X(t), t \in \mathbb{R}\} \) is a stationary stochastic process which satisfies conditions of Corollary 2.1 together with a \( f \)-wavelet \( \phi(y) \) and the corresponding \( m \)-wavelet \( \psi(y) \), \( f(y) \) is the spectral density of \( X(t), T > 0 \). If \( g(y) = \sqrt{f(y)} \) is bounded and absolutely continuous on \( \mathbb{R} \), \( \hat{\phi}(y), \hat{\psi}(y) \) are absolutely continuous and bounded on \( \mathbb{R} \), there exist derivatives \( g'(y), \hat{\psi}'(y), \hat{\phi}'(y) \) for all \( y \in \mathbb{R} \) and

\[ \sup_{y \in \mathbb{R}} |\hat{\psi}'(y)| \leq M < \infty, \]
\[ g(y) |\hat{\phi}(y)| \to 0 \text{ as } |y| \to \infty, \]
\[ g(y) |\hat{\psi}(y/2^j)| \to 0 \text{ as } |y| \to \infty, \quad j = 0, 1, \ldots, \]
\[ \int_{\mathbb{R}} |g'(y)| y^2 \, dy < \infty, \]
\[ \int_{\mathbb{R}} g(y) y^2 \, dy < \infty, \]
\[ \int_{\mathbb{R}} |g'(y)| |y| |\hat{\phi}(y)| \, dy < \infty, \]
\[ \int_{\mathbb{R}} g(y) |y| |\hat{\phi}(y)| \, dy < \infty, \]
\[ \int_{\mathbb{R}} g(y) |y| \, dy < \infty, \]
\[ \int_{\mathbb{R}} g(y) |y| |\hat{\psi}'(y)| \, dy < \infty, \]

then the following inequalities hold true:

\[ |\alpha_{00}(t_1) - \alpha_{00}(t_2)| \leq C_{a,0}|t_1 - t_2|, \quad (20) \]
\[ |\alpha_{0k}(t_1) - \alpha_{0k}(t_2)| \leq \frac{C_{a,1}}{|k|} |t_1 - t_2|, \quad k \neq 0, \quad (21) \]
\[ |\beta_{j0}(t_1) - \beta_{j0}(t_2)| \leq \frac{C_{g,0}}{2^{3/2}} |t_1 - t_2|, \quad (22) \]
\[ |\beta_{jk}(t_1) - \beta_{jk}(t_2)| \leq \frac{C_{\beta,1}}{2^{3/2}|k|} |t_1 - t_2|, \quad k \neq 0, \quad (23) \]

\( t_1, t_2 \in [0, T] \), where \( \alpha_{0k}(t), \beta_{jk}(t) \) are defined by (8) and (9), \( C_{a,0}, C_{a,1}, C_{\beta,0}, C_{\beta,1} \) are some constants.

**Proof:** To prove the statement we have to apply Lemma 3.1 with \( u(t, y) = g(y)\exp\{-iyt\}, \)
\[ v(y) = g(y)|y| \]
and
\[ w(y) = |g'(y)||y| + \sqrt{2} Tg(y)|y|. \]

**Theorem 3.2.** Suppose that a stochastic process \( X = \{X(t), t \in \mathbb{R}\} \) together with a \( f \)-wavelet \( \phi(y) \) and the corresponding \( m \)-wavelet \( \psi(y) \) satisfies conditions of Theorem 2.1 and Lemma 3.1, random
variables $\xi_{0k}, \eta_{jk}, j = 0, \infty, k \in \mathbb{Z}$, in expansion (4) are independent, $T > 0$. Then series (4) converges uniformly with respect to $t \in [0, T]$ with probability 1.

Proof. To prove the statement we have to recall that condition A holds for $\sigma(h) = h$ and apply Theorem 3.1 and Lemma 3.1.

An analogous statement holds for stationary processes.

**Corollary 3.1.** Suppose that a stationary stochastic process $X = \{X(t), t \in \mathbb{R}\}$ together with a f-wavelet $\phi(y)$ and the corresponding m-wavelet $\psi(y)$ satisfies the conditions of Corollary 2.1 and Lemma 3.2, random variables $\xi_{0k}, \eta_{jk}, j = 0, \infty, k \in \mathbb{Z}$, in expansion (7) are independent, $T > 0$. Then series (7) converges uniformly with respect to $t \in [0, T]$ with probability 1.

**Example.** It is easy to see that a centered Gaussian process $X = \{X(t), t \in \mathbb{R}\}$ with the correlation function

$$R(t, s) = \int_{\mathbb{R}} u(t, y) u(s, y) dy,$$

where

$$u(t, y) = \frac{1}{1 + t^2 + y^{2n}}, \quad n \geq 2, n \in \mathbb{N},$$

and an arbitrary Daubechies wavelet satisfy conditions of Theorem 3.2.

Indeed, for a Daubechies wavelet $\hat{\phi}(y), \hat{\psi}(y)$ are bounded on $\mathbb{R}$. The derivatives $\hat{\psi}'(y), \hat{\phi}'(y)$ exist and also are bounded on $\mathbb{R}$ (and therefore $\hat{\phi}(y)$ and $\hat{\psi}(y)$ are absolutely continuous). Obviously

$$\sup_{y \in \mathbb{R}} |u(t, y)| < \infty.$$

Since $u(t, y) \to 0$ as $|y| \to \infty$, we have

$$u(t, y)|\hat{\phi}(y)| \to 0 \quad \text{as} \quad |y| \to \infty,$$

$$u(t, y)|\hat{\psi}(y/2^j)| \to 0 \quad \text{as} \quad |y| \to \infty,$$

$t \in [0, T]$.

We have for $t_1, t_2 \in [0, T]$:

$$|u(t_1, y) - u(t_2, y)| = \frac{|t_1^2 - t_2^2|}{(1 + t_1^2 + y^{2n})(1 + t_2^2 + y^{2n})} \leq \frac{2T|t_2 - t_1|}{(1 + y^{2n})^2}.$$

Let us set

$$v(y) = \frac{2T}{(1 + y^{2n})^2}.$$

Then

$$|u(t_1, y) - u(t_2, y)| \leq v(y)|t_1 - t_2|, \quad t_1, t_2 \in [0, T],$$

and

$$\int_{\mathbb{R}} v(y) dy < \infty,$$

$$\int_{\mathbb{R}} v(y) |y| dy < \infty,$$
Consider $\Delta u' = |u'(t_1, y) - u'(t_2, y)|$ for $t_1, t_2 \in [0, T]$:

$$
\Delta u' = 2n|y|^{2n-1}\left| \frac{1}{(1 + t_1^2 + y^{2n})^2} - \frac{1}{(1 + t_2^2 + y^{2n})^2} \right|
= 2n|y|^{2n-1}\frac{|t_1^2 - t_2^2|}{(1 + t_1^2 + y^{2n})^2} \frac{(t_1^2 + t_2^2 + 2(1 + y^{2n}))}{(1 + t_2^2 + y^{2n})^2} \frac{1}{(1 + t_2^2 + y^{2n})^2} |t_2 - t_1|.
$$

Let us set

$$
w(y) = \frac{8nT|y|^{2n-1}(T^2 + 1 + y^{2n})}{(1 + y^{2n})^{4}},
$$

then

$$
|u'(t_1, y) - u'(t_2, y)| \leq w(y)|t_1 - t_2|, \quad t_1, t_2 \in [0, T],
$$

and

$$
\int_{\mathbb{R}} w(y)|y|dy < \infty,
\int_{\mathbb{R}} w(y)|\phi(y)|dy < \infty.
$$

So all the conditions of Theorem 3.2 are fulfilled.

4. Conclusions

Conditions of uniform convergence with probability 1 were obtained for a wavelet-based expansion of a second-order random process. This result is a counterpart of a theorem proved in Kozachenko and Turchin (2009) for stationary processes.

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References


