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Research Article

Modelling Marijuana Smoking Epidemics among Adults: An Optimal Control Panacea

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Abstract

Reducing the number of individuals involved in substance abuse in any community is usually a challenging problem. We consider the control of the spread of Marijuana smoking, one substance that is majorly abused, among adults. We propose a deterministic model for controlling the spread of Marijuana smoking incorporating education and awareness campaign as well as rehabilitation as control measures. We formulate a fixed time optimal control problem subject to the model dynamics with the goal of finding the optimal combination of the control measures that will minimize the cost of the control efforts as well as the prevalence of marijuana smoking in a community. We use Pontryagin's maximum principle to derive the optimality system and solve the system numerically. Results from our simulations are discussed.

Keywords: Substance abuse, Marijuana, Optimal control, Pontryagin's maximum principle, Optimality system, Hamiltonian

1. Introduction

Marijuana, also known as Indian hemp, grass, weed, reefer, harsh, pot, etc., is one of the world's most commonly abused illegal drugs [12]. It is a dry shredded green and brown mix of leaves, flowers, stems,and seeds from the hemp plant called Cannabis sativa. It is usually smoked in hand rolled cigarettes or in pipes. It is also smoked in blunts - cigars that have been emptied of tobacco and refilled with a mixture of marijuana and tobacco. However, some users mix it with food or brew as a tea. It is important to note that when Marijuana is smoked, the users starts to experience its effect almost immediately and the effect can last one to three hours. On the other hand, when Marijuana is eaten in food, it takes a while for the user to experience the effect, but it usually lasts longer.

The main psycho-active chemical in Marijuana is delta-9- tetrahydrocannabinol (THC). The THC acts on cannabinoid receptors which are found on neurons in many places in the brain. Specifically, the highest density of the cannabinoid receptors is found in parts of the brain that influence pleasure, memory, thinking, concentration, sensory and time perception, and coordinated movement [12]. Marijuana over-activates the endocannabinoid system; thus causing variety of adverse effects that users experience. These include distorted perceptions, impaired coordination, difficulty with thinking, and problem solving, and disrupted learning and memory.

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Marijuana smoking comes with grave consequences for the users and society at large. Findings from series of studies have shown that marijuana use more than doubles a driver's risk of being involved in an accident. A number of studies also established an association between chronic Marijuana use and mental illness. In addition, Marijuana smoking heightened individual's risks of lung infection while it also increases the user's risk of heart attack by more than four fold within first hour of smoking it. Worse still, a large prospective study showed that people who start smoking as teenagers experience significant decline in their intelligent quotient and that when this category of people eventually quit marijuana smoking as adults, the lost cognitive abilities were never restored [12, 16].

Some of the implications of high prevalence of Marijuana smoking in any society are increase in crime rate with its attendant upsurge in lost of lives and properties, more deaths due to crimes and accidents, pronounced loss of productive man working hours, reduction in people's contribution to the economy due to reduction in their potential as a result of marijuana use, and continuous enormous expenditure on rehabilitation of addicted users and prevention programmes [15]. In view of the monumental social and economical consequence of marijuana smoking on the users as well as the society, there is the need to evolve interventions that will help mitigate the continuous spread of the habit and also evaluate the impact of different combinations of these interventions.

It is against this background that we model the spread of marijuana smoking habits among adults using epidemiological contact model often adopted in the study of dynamics of social and behavioural processes [16]. For example, Kalula and Nyabadza [13] extended the model by White and Comiskey [21] to model substance abuse in South Africa in order to qualitatively investigate the dynamics of substance abuse and predict drug abuse trends. Using data for methamphetamine (MA) substance abuse in Western Cape province, their results suggested that the substance abuse epidemic can be reduced by intervention programmes targeted at light drug users and by increasing the uptake rate into treatment for those addicted. In addition, the model projected trends showed a steady decline in the prevalence of methamphetamine abuse until 2015. In a related work, Bhunu [1] studied and analyzed a model for the spread of alcoholism based on the Zimbabwean data. The model results showed that supporting and encouraging moderate alcohol drinkers to quit alcohol consumption will in the long run be more effective in curtailing the spread of alcoholism than singly targeting alcoholics only, though he averred that targeting both categories of alcohol drinkers will be the best strategy.

In recent times, optimal control theory has been used extensively in series of real-life applications. For instance, it was used in deriving control strategies for mitigating the spread of different infectious diseases [8, 18, 6, 2]. Also, the theory was used to evolve effective treatment strategies for the treatment of some deadly or terminal ailments/infections [9, 11, 4]. Here, we consider the control of the spread of marijuana smoking habits among adults based on a proposed dynamic deterministic model. In section two, we describe the proposed model and its parameters while we also establish that the model is mathematically well-posed. In section three, we formulated the modelled problem as an optimal control problem, derive the co-states equations, characterize the optimal controls and constitute the optimality system. In section four, we numerically solve the resulting optimality system and discuss our findings.

2. Model Formulation

Our proposed model subdivides the adults' population into susceptibles $S(t)$, the casual smokers $C(t)$, the addicted smokers $A(t)$, and the Removed $R(t)$ individuals compartments. The $S(t)$ comprises adults who do not smoke marijuana and have never smoked it; $C(t)$ are those who smoke marijuana occasionally but have not yet become addicted to it, $A(t)$ are those who are addicted to marijuana smoking; $R(t)$ are those adults who develop "`no-marijuana" smoking attitude due to intense exposure to wide spread campaign against smoking of marijuana and other related substances that are commonly abused, societal condemnation, good parental upbringing, religious indoctrination, or effective rehabilitation having hitherto been involved in marijuana smoking. It is assumed that these individuals in the R class maintain this attitude of resistance to marijuana smoking for the

rest of their lives. The total adults' population is then given by $N(t) = S(t) + C(t) + A(t) + R(t)$
Thus, the marijuana smoking epidemic dynamics within the youth population is given by the equations:

$$\begin{array}{rcl} \dot{S} &=& \Lambda - \frac{\beta_1 CS}{N} - \frac{\beta_2 AS}{N} - dS - u_1 S \\ \dot{C} &=& \frac{\beta_1 CS}{N} + \frac{\beta_2 AS}{N} - kC - u_2 C - dC \\ \dot{A} &=& kC - u_3 A - dA - \delta A \\ \dot{R} &=& u_1 S + u_2 C + u_3 A - dR \end{array} \tag{2.1}$$

and $N(t)$ satisfies the equation

$$\dot{N} = \Lambda - Nd - \delta A$$

The parameters for the model are defined in Table 2.1 below

Table 2.1. Description of parameters used in the model

Parameter	Description
Λ	recruitment rate into S class
d	natural removal rate due to age or death
β_1	the smoking-habit spread rate for prolong contacts with C class
β_2	the smoking-habit spread rate for prolong contacts with A class $(\beta_1 < \beta_2)$
k	progression rate from C class into A class
u_1	effective removal rate from the S class due to societal, media, and parental influence per unit time.
u_2	effective rehabilitation rate for the C class per unit time
u_3	effective rehabilitation rate for the A class per unit time
δ	Marijuana smoking induced -death rate for the addicts

Note that $\lim_{t\to\infty} N(t) \leq \frac{\Lambda}{d}$. However, under the dynamics described by (2.1), the region Ω defined by

$$\Omega = \{(S, C, A, R) \in \mathbb{R}^4_+ \mid S + C + A + R \leq \frac{\Lambda}{d}\},$$

is positively invariant.

Lemma 2.1. The region $\mathbb{R}^4_+$ is positively-invariant for the model (2.1) (i.e. the model does not predict negative values for the states variables at any future time).

Proof
Let $t_1 = sup\{t > 0 | S \geq 0,\ C \geq 0,\ A \geq 0,\ R \geq 0,\ \in [0, t]\}$. From 2.1, we have:

$$\frac{dS}{dt} = \Lambda - (\lambda(t) + d + u_1(t))S, \qquad where\ \lambda(t) = \frac{\beta_1 C + \beta_2 A}{N}.$$

This is same as

$$\frac{dS}{dt} + (\lambda(t) + d + u_1(t))S = \Lambda$$

and this implies that

$$\frac{d}{dt}\big(S(t) exp\{dt + \int_0^t (\lambda(\tau) + u_1(\tau))d\tau\}\big) = \Lambda\, exp\{dt + \int_0^t (\lambda(\tau) + u_1(\tau))d\tau\}.$$

Thus,

$$S(t_1)\, exp\{dt_1 + \int_0^{t_1} (\lambda(\tau) + u_1(\tau))d\tau\} - S(0) = \int_0^{t_1} \Lambda\, exp\{d\psi + \int_0^{\psi} (\lambda(\epsilon) + u_1(\epsilon))d\epsilon\}d\psi$$

Hence,

$$\begin{aligned} S(t_1) &= S(0)\, exp\{-\big(dt_1 + \textstyle\int_0^{t_1} (\lambda(\tau) + u_1(\tau))d\tau\big)\} \\ &\quad + exp\{-\big(dt_1 + \textstyle\int_0^{t_1} (\lambda(\tau) + u_1(\tau))d\tau\big)\} \times \textstyle\int_0^{t_1} \Lambda\, exp\{d\psi + \int_0^{\psi} (\lambda(\epsilon) + u_1(\epsilon))d\epsilon\}d\psi \\ &\geq 0 \end{aligned} \tag{2.2}$$

Similarly, we can show that $C(t) \geq 0$, $A(t) \geq 0$, and $R(t) \geq 0$. This complete the proof. □
The above lemma is important because it guarantees that the model variables are continuously biologically meaningful, since population size can not be negative.

Lemma 2.2. The region Ω is an attractor and it attracts all solutions starting in the interior of the positive orthant $\mathbb{R}^4_+$

Proof
We use the non-negativity of the model state variables established in the preceding lemma and

$$\dot{N} = \Lambda - dN - \delta A;$$

for initial conditions in $\mathbb{R}^4_+$ and $t \geq 0$, to obtain $\dot{N} \leq \Lambda - dN$. This implies that

$$\frac{d}{dt}\big(Ne^{dt}\big) \leq \Lambda e^{dt} \Rightarrow N(t)e^{dt} - N(0) \leq \frac{\Lambda}{d}(e^{dt} - 1) \leq \frac{\Lambda}{d}e^{dt}.$$

So for all $t \geq 0$,

$$N(t) \leq N(0)e^{-dt} + \frac{\Lambda}{d} \tag{2.3}$$

If (S^*, C^*, A^*, R^*) is an Ω limit point of an orbit in $\mathbb{R}^4_+$, then there is a subsequence $t_i \to \infty$ such that

$$\lim_{i\to\infty} (S(t_i), C(t_i), A(t_i), R(t_i)) = (S^*, C^*, A^*, R^*).$$

Hence,

$$\lim_{i\to\infty} N(t_i) = N^* = S^* + C^* + A^* + R^*.$$

From 2.3 (by evaluation at $t = t_i$ and passing to the $limit\ i \to \infty$), it follows that $N^* \leq \frac{\Lambda}{d}$ and hence that $(S^*, C^*, A^*, R^*) \in \Omega$. □

Thus, for any initial starting point $(S_0,\ C_0,\ A_0,\ R_0) \in \mathbb{R}^4_+$, the trajectory lies in Ω. Therefore, the system is both mathematically and epidemiologically well-posed.

3. Optimal Control

We define our objective functional as

$$\mathbb{Z} = \min_{u_1,u_2,u_3} \int_0^{t_f} \big(\frac{w_1}{2}u_1^2 + \frac{w_2}{2}u_2^2 + \frac{w_3}{2}u_3^2 + w_4 C + w_5 A\big)dt \tag{3.1}$$

subject to system of equations (2.1) with appropriate states initial conditions and t_f is the final time while the control set $\mathbb{U}$ is Lebesgue measurable and it is defined as

$$\mathbb{U} = \{(u_1(t), u_2(t), u_3(t))|0 \leq u_i \leq u_{imax} < 1,\ i = 1, 2, 3,\ t \in [0, T]\} \tag{3.2}$$

and the weight constants w_1, w_2, w_3, w_4, w_5 are the relative weights and helps to balance each terms in the integrand so that any of the terms does not dominate. Here, it is important to note that $w_1,\ w_2,\ w_3$ are relative measures of the cost or effort required to implement each of the associated controls while $w_4,\ w_5$ are relative measures of the importance of reducing the associated classes on the spread of marijuana smoking habits in the target community.

The upper bound for each of the controls $(u_{1max},\ u_{2max},\ u_{3max})$ will depend on the budget allocated for the execution of each of the control measures. For instance, we shall hypothetically set $u_{1max} = 0.7$, $u_{2max} = 0.7$, and $u_{3max} = 0.7$ in our subsequent simulations. We wish to determine the optimal combination of controls u_1, u_2, and u_3 that will be adequate to minimize cost of the education/awareness campaigns together with the cost of rehabilitation as well as reduce the prevalence of the marijuana smoking habits over a fixed time period.

3.1 *Existence of an optimal control strategy for the problem*

Here, we examine the sufficient conditions for the existence of a solution to the optimal control problem.

Theorem 3.1. There exists an optimal control set $(u_1^*,\ u_2^*,\ u_3^*)$ with corresponding solution (S^*, C^*, A^*, R^*); to the model system (2.1)), that minimize $\mathbb{Z}$ over $\mathbb{U}$.

Proof. The existence of the optimal control is guaranteed by the compactness of the control and the state space and the convexity in the problem based on Theorem 4.1 of Chapter III and its corresponding Corollary in [5]. The following non-trivial requirements from Fleming and Rishel's theorem are stated and verified below:

(1) The set of all solutions to the (2.1) and its associated initial conditions together with the corresponding control functions in $\mathbb{U}$ is non empty

(2) The state system can be written as a linear function of the control variables with coefficients dependent on time and state variables.

(3) The integrand L in equation (3.1) is convex on $\mathbb{U}$ and additionally satisfies $L(t, S, C, A, R, u_1, u_2, u_3) \geq c_1|(u_1, u_2, u_3)|^{\alpha} - c_2$, where $c_1, c_2 > 0$ and $\alpha > 1$.

We refer to Theorem 3.1 by Picard-Lindelof in [3]. Based on this theorem, if the solutions to the state equations are a priori bounded and if the state equations are continuous and Lipschitz in the state variables, then there is exist a unique solution corresponding to every admissible control set in $\mathbb{U}$. Using the fact that for all $(S, C, A, R) \in \Omega$, all the model states are bounded below and above; then the solutions to the state equations are bounded. In addition, it is direct to show the boundedness of the partial derivatives with respect to the state variables in the system and this shows that the system is Lipschitz with respect to the state variables. Thus, the condition 1 holds.

As we can observe from the state equations (2.1), the state equations are linearly dependent on the controls $u_1,\ u_2$, and u_3. Thus, condition 2 also holds.

To establish condition 3, we observe that the integrand L in our objective functional is convex since it is quadratic in the controls. We only then need to prove the bound on L. This is shown as below:

$$\begin{aligned} L &= \tfrac{1}{2}(w_1u_1^2 + w_2u_2^2 + w_3u_3^2) + w_4C + w_5A \\ &\geq \tfrac{1}{2}(w_1u_1^2 + w_2u_2^2 + w_3u_3^2) \qquad since\, w_i > 0\; i = 1..5 \\ &\geq \tfrac{1}{2}(w_1u_1^2 + w_2u_2^2 + w_3u_3^2) - w_1 \qquad since\, w_1u_1^2 - w_1 \leq 0 \\ &\geq min(\tfrac{1}{2}w_1, \tfrac{1}{2}w_2, \tfrac{1}{2}w_3)(u_1^2 + u_2^2 + u_3^2) - w_1 \\ &\geq W||(u_1, u_2, u_3)||^2 - w_1 \quad where\, W = min(\tfrac{1}{2}w_1, \tfrac{1}{2}w_2, \tfrac{1}{2}w_3) \end{aligned} \tag{3.3}$$

The above then established a bound on L. Thus, we have a unique solution of the optimality system for small time interval due to the opposite time orientations of the state equations and the adjoint equations. Moreover, the uniqueness of the solution of the optimality system guarantees the uniqueness of the optimal control if it exists. □ □

3.2 *Characterization of the optimal controls*

We characterize the optimal controls $u_1^*,\ u_2^*,\ u_3^*$ which gives the optimal levels for the various control measures and the corresponding states (S^*, C^*, A^*, R^*). The necessary conditions for the optimal controls are obtained using the Pontryagin's maximum principle [14] while the terminal conditions on the adjoint variables are obtained based on the transversality condition [7].

Theorem 3.2 (Necessary conditions)*.* Let $(u_1^*, u_2^*, u_3^*) \in \mathbb{U}$ be an optimal control with the corresponding states S^*, C^*, A^*, and R^*. Then, there exist the adjoint variables λ_i for $i = 1...4$ which satisfy:

$$\begin{aligned} \lambda_1' &= \lambda_1(u_1 + d) + (\lambda_1 - \lambda_2)\big(\tfrac{\beta_1 C + \beta_2 A}{N}\big) - (\lambda_1 - \lambda_2)\big(\tfrac{\beta_1 CS + \beta_2 AS}{N^2}\big) - \lambda_4 u_1 \\ \lambda_2' &= -w_4 + (\lambda_1 - \lambda_2)\big(\tfrac{\beta_1 S}{N}\big) - (\lambda_1 - \lambda_2)\big(\tfrac{\beta_1 CS}{N^2}\big) + \lambda_2(k + u_2 + d) - \lambda_3 k - \lambda_4 u_2 \\ \lambda_3' &= -w_5 + (\lambda_1 - \lambda_2)\big(\tfrac{\beta_2 S}{N}\big) - (\lambda_1 - \lambda_2)\big(\tfrac{\beta_2 AS}{N^2}\big) + \lambda_3(u_3 + d + \delta) - \lambda_4 u_3 \\ \lambda_4' &= \lambda_4 d \end{aligned} \tag{3.4}$$

and the transversality conditions

$$\lambda_i(t_f) = 0,\ for\ i = 1..4. \tag{3.5}$$

with the optimal controls defined as follows

$$\begin{aligned} u_1^* &= min\,\{max(0,\ \tfrac{S(\lambda_1 - \lambda_4)}{w_1}), u_{1max}\}, \\ u_2^* &= min\,\{max(0,\ \tfrac{C(\lambda_2 - \lambda_4)}{w_2}), u_{2max}\}, \\ u_3^* &= min\,\{max(0,\ \tfrac{A(\lambda_3 - \lambda_4)}{w_3}), u_{3max}\}. \end{aligned}$$

Proof. Using Pontryagin's maximum principle, we obtain (3.4) from

$$\lambda_1' = -\frac{\partial \mathbb{H}}{\partial S},\ \lambda_2' = -\frac{\partial \mathbb{H}}{\partial C},\ \lambda_3' = -\frac{\partial \mathbb{H}}{\partial A},\ \lambda_4' = -\frac{\partial \mathbb{H}}{\partial R}, \tag{3.6}$$

where the Hamiltonian $\mathbb{H}$ is given by

$$\begin{aligned} \mathbb{H} = &\tfrac{w_1}{2}u_1^2 + \tfrac{w_2}{2}u_2^2 + \tfrac{w_3}{2}u_3^2 + w_4C + w_5A \\ &+\lambda_1(\Lambda - \tfrac{\beta_1 CS}{N} - \tfrac{\beta_2 AS}{N} - dS - u_1S) \\ &+\lambda_2(\tfrac{\beta_1 CS}{N} + \tfrac{\beta_2 AS}{N} - kC - dC - u_2C) \\ &+\lambda_3(kC - u_3A - dA - \delta_1 A) \\ &+\lambda_4(u_1S + u_2C + u_3A - dR) \end{aligned} \tag{3.7}$$

The transversality condition have the form (3.5), since all the states are free at the terminal time.

The Hamiltonian is maximized with respect to the controls at the optimal control $u^* = (u_1^*, u_2^*, u_3^*)$, thus we differentiate $\mathbb{H}$ with respect to $u_1, u_2,$ and u_3 on $\mathbb{U}$ respectively to obtain

$$\begin{aligned} \frac{\partial \mathbb{H}}{\partial u_1} &= w_1 u_1 - \lambda_1 S + \lambda_4 S = 0 \; at \; u_1 = u_1^* \\ \frac{\partial \mathbb{H}}{\partial u_2} &= w_2 u_2 - \lambda_2 C + \lambda_4 C = 0 \; at \; u_2 = u_2^* \\ \frac{\partial \mathbb{H}}{\partial u_3} &= w_3 u_3 - \lambda_3 A + \lambda_4 A = 0 \; at \; u_3 = u_3^* \end{aligned} \tag{3.8}$$

Hence, solving for u_1^*, u_2^*, and u_3^* on the interior sets gives:

$$\begin{aligned} u_1^* &= \frac{S(\lambda_1 - \lambda_4)}{w_1}, \\ u_2^* &= \frac{P(\lambda_2 - \lambda_4)}{w_2}, \\ u_3^* &= \frac{A(\lambda_3 - \lambda_4)}{w_3}. \end{aligned} \tag{3.9}$$

We can now impose the bounds $0 \le u_1 \le u_{1max}$, $0 \le u_2 \le u_{1max}$, and $0 \le u_3 \le u_{1max}$ on the controls to get

$$\begin{aligned} u_1^* &= min\,\{max(0, \frac{S(\lambda_1 - \lambda_4)}{w_1}), u_{1max}\}, \\ u_2^* &= min\,\{max(0, \frac{P(\lambda_2 - \lambda_4)}{w_2}), u_{2max}\}, \\ u_3^* &= min\,\{max(0, \frac{A(\lambda_3 - \lambda_4)}{w_3}), u_{3max}\}. \quad \square \end{aligned}$$

□

It is important to note that the above controls characterization can be written in the simpler piecewise form below:

$$u_1^* = \begin{cases} 0 & \text{when } \lambda_1 - \lambda_4 < 0 \\ min\{\frac{S(\lambda_1 - \lambda_4)}{w_1}, u_{1max}\} & \text{when } \lambda_1 - \lambda_4 > 0 \end{cases} \tag{3.10}$$

$$u_2^* = \begin{cases} 0 & \text{when } \lambda_2 - \lambda_4 < 0 \\ min\{\frac{P(\lambda_2 - \lambda_4)}{w_2}, u_{2max}\} & \text{when } \lambda_2 - \lambda_4 > 0 \end{cases} \tag{3.11}$$

$$u_3^* = \begin{cases} 0 & \text{when } \lambda_3 - \lambda_4 < 0 \\ min\{\frac{A(\lambda_3 - \lambda_4)}{w_3}, u_{3max}\} & \text{when } \lambda_3 - \lambda_5 > 0 \end{cases} \tag{3.12}$$

We can now solve the optimality system which consists of the state system (2.1) with its associated initial conditions and the adjoint system (3.4) with its transversality conditions coupled with the controls characterization (3.10), (3.11), (3.12).

4. Estimation of Parameters

The recruitment rate (Λ) into the susceptible was estimated using the average annual increase in the Nigeria's population for age 15-59 years. Based on the United Nations population prospects for age 15-59 years from year 1990 -2010, we estimate the adult population constant annual increase to be 1.701 million per year [20].

The life expectancy of people in Nigeria is about 55 year [19]. We assumed the adult stage starts at age 15 years, and adjusted the life expectancy with the starting age at the adult stage, so we estimate $d = \frac{1}{55-15} = 0.025\, yr^{-1}$.

Considering the fact that Marijuana users have 4.8 fold increase risk of heart attack in the first hour after smoking the drug [12]. Also, Marijuana use more than doubles a driver's risk of being involved in an accident [12, 22] . Thus, we estimate the Marijuana-induced death rate (δ) to be the average $\frac{1}{2}(4.8 + 2.0)\times$ natural death rate. This gives $\delta = 3.4 \times d = 0.085\, yr^{-1}$.

Using the information that one in every six individuals who start smoking at their youthful age eventually become addicted [12] and complementing this with the time spent in this casual smoking class; we estimate $k = \frac{a}{1-a}(d + u_2)$ where a is the proportion of the Marijuana smokers that eventually get addicted [13]. Thus, we have $k \in [0.005, 0.205]$.

The prevalence of marijuana smoking among Nigerian youth is about 9.4% [23] while the prevalence in the entire population is 5.0% [22]. However, the prevalence of Marijuana smoking among adults in Nigeria varies from one location to another. The prevalence could be as high as 26% in places like Port-Harcourt , Rivers state [23]; it may be as low as 5% in places like Ilorin, Kwara state [22]; while it is relatively moderate in places like Zaria, Kaduna state with a prevalence rate of 9.4% [23]. Here, we assume that the prevalence of Marijuana smoking among Nigerian adults is about 10%. Thus, we estimate the smoking-habit spread rate as the product of the probability that sufficient contact between the a Marijuana smoker and a susceptible results in Marijuana smoking for the susceptible individual, and the probability of getting in contact with a Marijuana smoker in Nigeria. In addition, we assume that the probability of transferring the habit by an addicted individual is 1 while the probability of transferring the habit by a casual smoker is 0.5. Therefore, we estimate $\beta_2 = 0.1$ and $\beta_1 = 0.05$.

Table 4.1. Estimated Parameter values

Parameter	Estimated value	Sources
Λ	1.701	Estimate and [20]
d	0.025	Estimate and [19]
β_1	0.05	Estimate
β_2	0.1	Estimate
k	$[0.005, 0.205]$	Estimate and [12]
u_1	$[0.0, 1.0]$	Estimate
u_2	$[0.0, 1.0]$	Estimate
u_3	$[0.0, 1.0]$	Estimate
δ	0.085	Estimate and [12, 22]

5. Simulation Results and Discussion

We solved the resulting optimality system numerically using a forth order iterative Runge-Kunta scheme. This method solves the state equation with an initial guess for u_1, u_2, and u_3 forward in time; after which it solves the adjoint equations backward in time while the controls are continuously updated based on control equations (3.10 - 3.11). The computational procedure is done iteratively until results converges (See [10]). Using Nigeria's adult population data for year 2010, with the assumption that only 10% of the population smoke marijuana and that one out of every six smokers is an addict, we obtain the initial population for each of the compartments as $S(0) = 73.94$, $C(0) = 6.85$, $A(0) = 1.37$ and $R(0) = 0.0$. We simulated the optimality system with parameter values from Table(4.1), weight constants $w_1 = 1.0$, $w_2 = 10.0$,$w_3 = 100.0$, $w_4 = 1.0$,$w_5 = 1.0$; and we assume that the resources available will only be able to implement the following maximum levels of the controls: $u_1 = 0.7$, $u_2 = 0.7$, and $u_3 = 0.7$. We investigated three scenarios: the situation where the only control measures employed is rehabilitation (i.e. $u_1 = 0.0$); the situation where only behavioural change due to education/enlightenment cum high moral values is employed as the control measure (i.e $u_2 = u_3 = 0.0$); the situation where both rehabilitation and behavioural change control measures are employed.

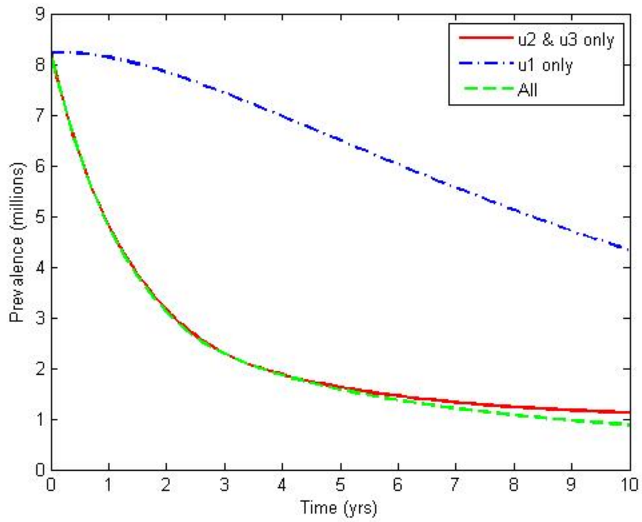

Fig. 5.1. The prevalence of marijuana smoking in the society for each of the three scenarios.

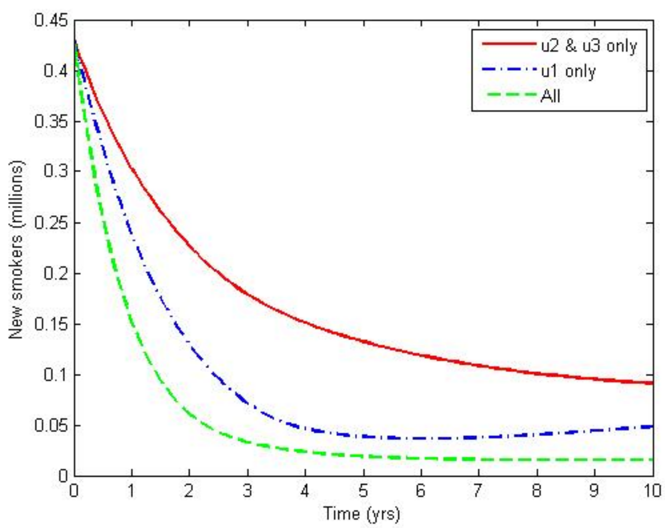

Fig. 5.2. The New cases of marijuana smoking in the society for each of the three scenarios.

Figures 5.1 and 5.2 show the prevalence and New cases of Marijuana smoking among adult in Nigeria for the three different scenarios. We observe that the deployment of rehabilitation as the only control measure resulted in a remarkable reduction in the prevalence of the habit when compared to the impact of behavioral change control, although a combination of the two measures is more effective. However, the use of the rehabilitation control leads to more new cases of the habit while the use of the behavioural change control yielded relatively less new cases of the habits. This suggests that the two measures should be deployed together in order to reduce both the prevalence and the new cases of the habit concurrently.

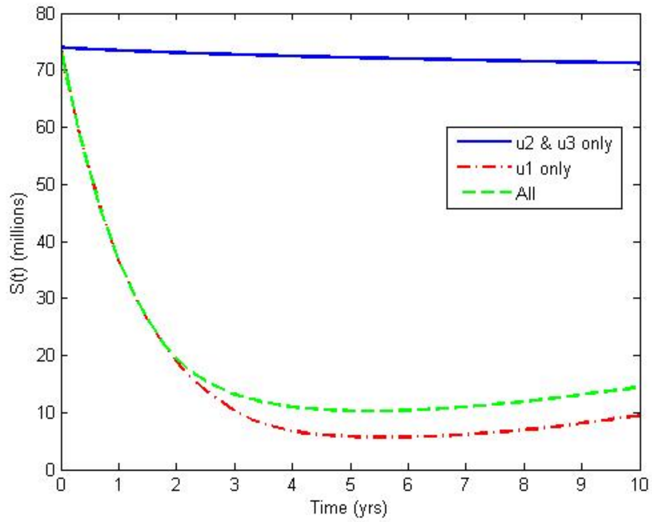

Fig. 5.3. Population of the Susceptible class for each of the three scenarios.

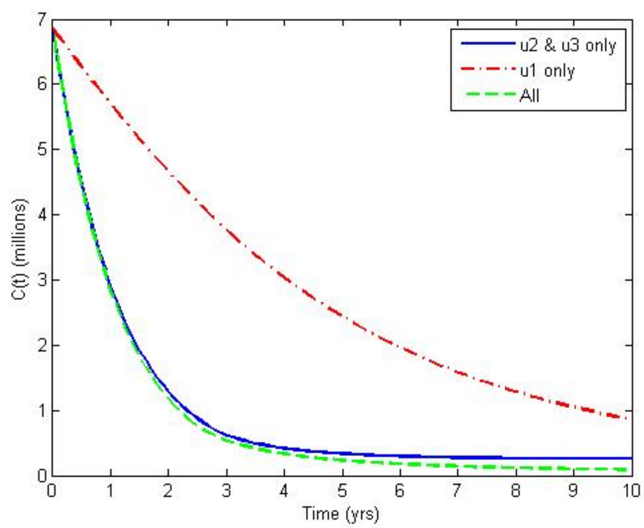

Fig. 5.4. Population of the Casual smokers' class for each of the three scenarios.

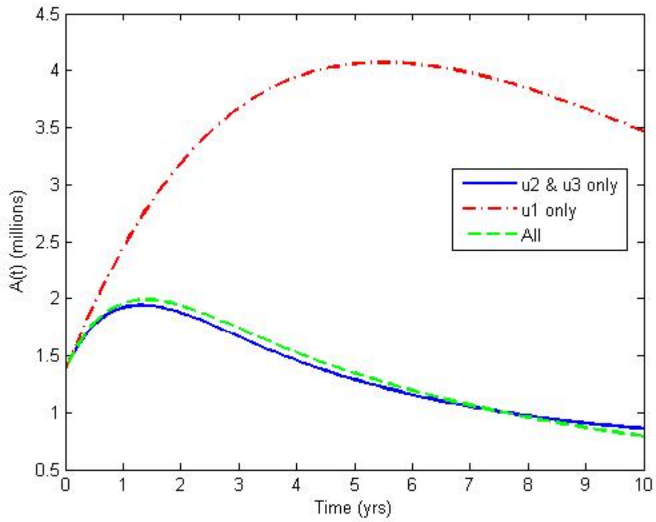

Fig. 5.5. Population of the Addicted smokers' class for each of the three scenarios.

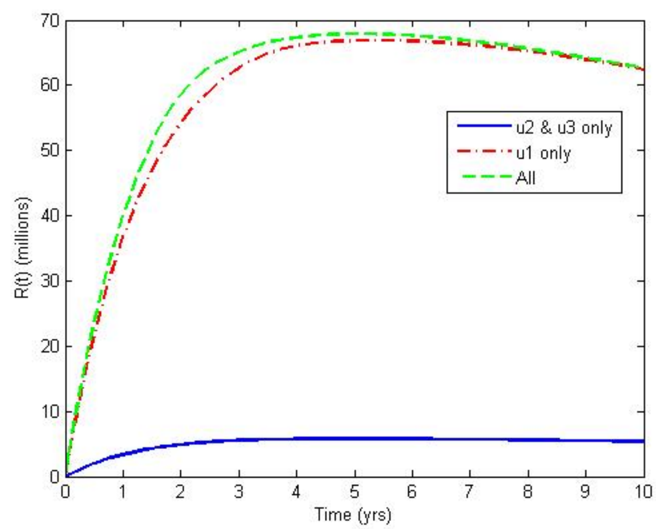

Fig. 5.6. Population of the Removed and Recovered class for each of the three scenarios.

The population profiles of our model's different compartments are displayed in Figures 5.3, 5.4, 5.5, and 5.6. As expected, the impact of the behavioural change control is more pronounce on the population of Susceptible and removed/recovered classes while the impact of the rehabilitation control is more effective on the population of the Casual and addicted smokers classes. Nevertheless, either of the two controls has impact on the population of each of the compartment at any given time. Thus, corroborating the fact that the two control measures should be deployed concurrently, even if at different level of effectiveness.

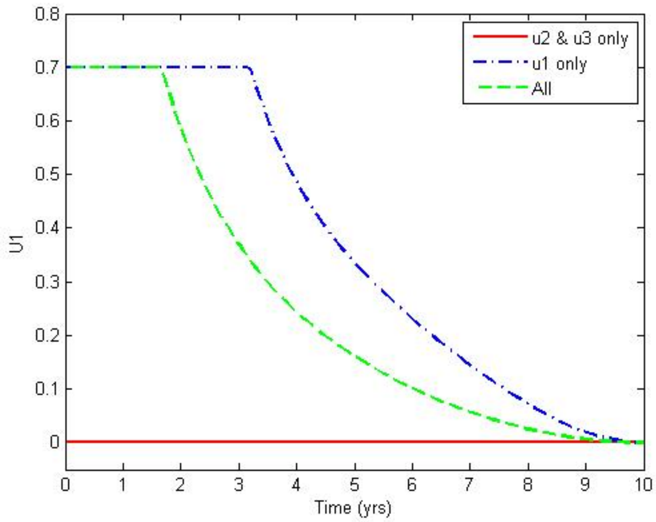

Fig. 5.7. Profile of Control u_1 for each of the three scenarios

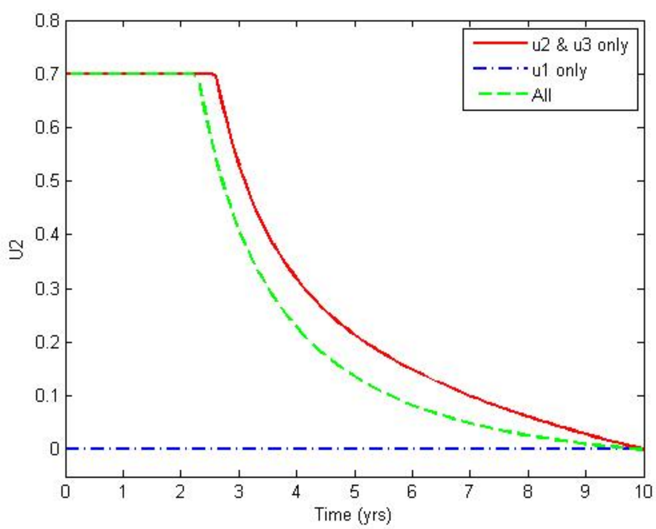

Fig. 5.8. Profile of Control u_2 for each of the three scenarios

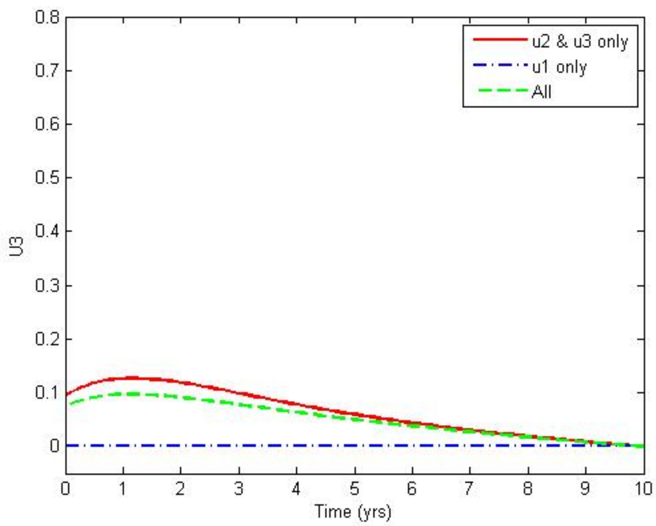

Fig. 5.9. Profile of Control u_3 for each of the three scenarios

The levels of effectiveness of each of the control measures for the three scenarios were displayed in Figures 5.7, 5.8, and 5.9. The graphs show that for concurrent reduction in both the prevalence and New cases of Marijuana smoking among adults both control measures - Behavioural change and Rehabilitation - has to be deployed. Moreover, the levels of effectiveness required of each of the three control measures is significantly lesser than that in which only either of the two control measures is used. This may in turn reduce the cost of implementing both measures concurrently. Nevertheless, the results portray that a relatively less effectiveness is require for the rehabilitation of the addicted class which indirectly infers that emphasis should be placed on rehabilitating of casual smokers before they get addicted and the susceptibles inculcating the right behaviour due to exposure to education /enlightenment campaign cum development of high moral values to enable them consistently resist smoking tendencies as they continuously mingle with smokers.

6. Concluding Remarks

We presented a deterministic model for controlling Marijuana smoking among adults. We demonstrated that the model is mathematically well posed. Thereafter, we formulated an optimal control problem subject to the model dynamics with positive change in susceptible individual's behaviour due to education/enlightenment campaign cum inculcation of good moral values, and the rehabilitation of the smokers as control measures.

We proved the existence and uniqueness of the optimal control and characterize the the controls using Pontryagin's Maximum principle. We solved the resulting optimality system numerically and the results showed that the optimal way to curtail the spread of Marijuana smoking habits among adults is to concurrently deploy the change in behaviour control for the susceptibles and the rehabilitation control for the smokers in order to reduce both the prevalence and the new cases of the habit. This dual measures is much effective than deploying either of the two measures singly. since it leads to a continuous reduction in the prevalence and new cases of the habit while the level of effectiveness of each of the control required is continuously reduced over time.

However, it will be worthy to mention here that the unavailability of adequate data may affect the results from this work, since the contrary would have helped in better estimates of the parameters and also help in comparing the predictions of the model with real life data. Also, the ban on Marijuana smoking and its abuse resulting in under reporting of the cases may actually affect the correctness of the data collected on the prevalence and new cases of the habits. Nevertheless, the model gives good predictions of the Marijuana smoking prevalence and new cases within the context of the study.

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