Research Article

Deployment of Long Flexible Element on Spacecraft with Magnetic Damper

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Abstract

The present paper deals with the study of the dynamics of a spacecraft with a gravitational system of stabilization, with a magnetic damper, and pitch flywheel. Primarily the boom of the gravitational stabilizer is a prestressed tape wound on a special drum. When the drum starts deploying the tape, its takes the shape of an elastic long boom with the mass on its tip. The objective of the study is the mechanical and mathematical modeling and numerical simulation of the spacecraft dynamics. The equations of motion are derived with the use of the Eulerian-Lagrangian formalism. The information, which is obtained by a numerical simulation, gives the conclusion that the deployment and functioning of the spacecraft can be used to design a rational choice of parameters of the spacecraft. The data obtained allow choosing of the most suitable law of deployment and parameters of the magnetic damper. A detailed simulation has allowed analyzing the dynamic behavior of the design at various values of parameters of the spacecraft with the gravitational stabilizer. Data obtained facilitate selection the most appropriate deployment, structure and damper parameters.

Keywords: Dynamics of spacecraft; Magnetic damper; Gravitational stabilizer; Modeling and numerical simulation

1. Introduction

Various systems of orientation and stabilization of spacecraft have been applied depending on tasks in view. They can be divided into passive, active and combined ones. Such systems should provide a tendency of a frame of reference fixed in the spacecraft to a certain attitude or law of a three-dimensional motion, and provide dissipation of oscillation energy of spacecraft. One example of such a spacecraft is the gravitationally stabilized spacecraft. Various damping devices have been used for dissipation of energy in oscillations with respect to the local vertical. Their main principle consists in coupling of the spacecraft with the additional body that performs a relative rotary

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motion in a viscous environment. Two degree gyro-dampers are established in certain cases on such spacecraft to reduce amplitude of oscillations about the mass center.

During perfection of dampers and search of new ways of vibration suppression of gravitational spacecraft, particular attention has been given to questions of use of the magnetic field of the Earth. As a result, the spherical magnetic damper that was described by Alper and O'Neill (1967) and Sokolov (1973) had been created by the General Electric company. Its basis is a constant magnet that is placed in two concentric spheres separated by a liquid layer. The outside sphere is connected with the spacecraft; internal one contains a constant magnet. Energy dissipation occurs due to viscosity of a liquid and the vortex currents induced by the field of the mobile magnet in the shell of the outside sphere.

Such dampers can be placed at any point of the spacecraft where the magnet presence does not hamper normal activity of spacecraft devices. So, Sadov (1969) studied the dynamics of the gravitational spacecraft with the magnetic damper in the spacecraft body, however, it is the most expedient to place the damper on the tip of the gravitational boom, using it simultaneously for modifying of the spacecraft inertia tensor.

The inertia ellipsoid of the gravitationally stabilized spacecraft is a rotation ellipsoid, as a rule. Taking it into attention it is the most convenient way to supply the yaw orientation using a handwheel that rotates with constant angular velocity about the pitch axis. At this restoring torque created by the handwheel has effect upon the roll and yaw stabilization as it was shown by Crespoda Silva Marcelo (1969).

Among possible factors, making the most impact on the accuracy of stabilization of the gravitational spacecraft with the magnetic damper and pitch handwheel, it is necessary to emphasize the interaction of the magnetic damper with flexible spacecraft structural elements. This problem did not find a proper attention in scientific publications. This paper deals with studying the dynamics of the gravitationally stabilized spacecraft that has the magnetic damper positioned on the tip of the flexible gravitational stabilizer boom for dissipation of energy of angular oscillations. The study of such structures is required for minimization of deployment duration, mass, and power resources, for analysis of the effect of such devices on the spacecraft attitude motion.

2. Physical Model of System

The magnetic damper is uncaged after the spacecraft separation from the last stage of the launcher and after pre-damping of the spacecraft. It reduces the components of the spacecraft angular velocity approximately to magnitude of the orbital angular velocity during eight hours. The gravitational stabilizer is deployed for a certain time calculated on the onboard computer after receiving the signal of the gauge of the Earth presence.

Example of the mechanism of deployment of the gravitational stabilizer boom is shown in Fig. 1. Here element 1 is the elastic core of the boom formed of the preliminary strained tape, elements 2 is the directing rollers, element 3 is the tape that has been reeled up on a drum, element 4 is the mechanism of the tape unwinding, element 5 is the case of the deployment device, and element 6 is the mass on the boom tip.
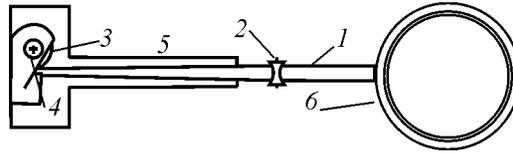
Fig. 1. Basic elements of the deployment system.

The deployed structures, especially those made of cores of the open cross-section, have essential flexibility. It imposes certain restrictions on parameters of the law of their deployment.

The magnetic damper is placed in the case with a spherical cavity. The spherical body with a rigid magnet is placed in it with a backlash. The centering of the internal sphere in external one is provided with repulsive action of the auxiliary horseshoe-shaped magnets from a bismuth layer. The linear magnet of the internal sphere interacts with a magnetic field of the Earth like the arrow of a compass turning to a local magnetic power line; this leads to the relative motion of the internal and external spheres. Dissipation of energy of spacecraft rotary motion is carried out due to losses at a viscous friction in liquid and owing to Foucault currents generated by the magnetic field of the linear magnet of the internal sphere in the metal case of the external sphere due to the relative motion. The case is made of non-magnetic material so that the vector of intensity of the geomagnetic field can guide the magnet. Thus, the magnetic damper represents in essence an additional body connected with the spacecraft only by the fictitious three-degree hinge. Such a damper is expedient to use as mass on the gravitational stabilizer tip since it has its own magnetic field, which can interact with the electronic devices located in the main spacecraft module, and with a magnetic field of this module. Removal of the magnetic damper on the gravitational stabilizer length allows reducing such an interaction. A pitch flywheel is installed often on such spacecraft for reduction of orientation errors: it is the flywheel rotating with constant angular velocity with the axis that is oriented parallel to the pitch axis.

3. Mechanical Model of System

The mechanical model of the system under consideration is presented as a main rigid body and body of variable configuration attached to it (Fig. 2). The body S_1 is the gyrostatic part, which does not change the rotational body inertia. The motion of the body S_1 is determined by the velocity vector $\vec{v}_{C_1}$ of the point C_1 with respect to the mass center C and by the vector of absolute angular velocity $\vec{\omega}$ of the body S_1. The body S_2 consists of a flexible boom of variable length with the point mass on its tip, and the system of the gravitational stabilizer deployment, which is shown in Fig. 1. The following frames of reference that are shown in Fig. 2 are useful for the problem statement: $\overline{CXYZ}$ is the earth-centered inertial reference frame; C_1xyz is the body S_1 fixed reference frame with origin at its mass center; C_1z is directed along design position of the gravitational stabilizer axis. The orbital frame of reference $Cx^{or}y^{or}z^{or}$ that is fixed in instant position of the mass center is introduced with the standard directions of its axes. The body S_2 is a deployed part with C_2 as the instant mass center and does not contain gyrostatic components (for example, mass of the drum for the tape; drum rotation can be considered for calculation of the relative moment of momentum).
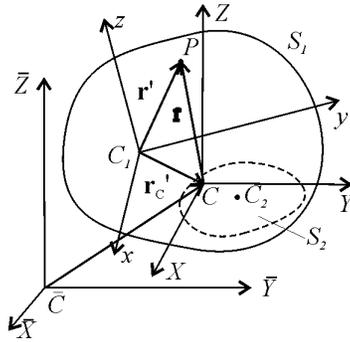
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Fig. 2. Generalized mechanical model of spacecraft with deployable antenna.

The position vector $\vec{r}$ defines the location of the arbitrary point P with respect to the reference frame $\overline{CXYZ}$, and the position vector $\vec{r}\,'$ – with respect to the reference frame $C_1 xyz$. In contrast to the problem of the relative motion of attached bodies described by Lurie (2002), the expression for $\vec{r}\,'$ depends on time t explicitly, not only through the generalized coordinates: $\vec{r}\,' = \vec{r}\,'(q_1, \ldots, q_n, t)$ since the deployment of the tape takes place in accordance with the time dependent law. As a result, $\vec{r}\,'_i$ varies during the deployment even in the absence of the relative elastic motion of the design.

Relative deflections of the magnetic damper mass center on the boom tip in directions of the axes $C_1 x$ and $C_1 y$ are chosen further as the generalized coordinates, which determine the relative motion of the system (Fig. 3).

Since angular motion of the magnetic damper central body is not connected kinematically with motion of its external case, it should be considered as a free rigid body from the point of view of the attitude motion. Its motion may be described by the Euler-Lagrange equations. It is necessary to introduce also the frame of reference $C_D x_D y_D z_D$ fixed in the mass center of the internal magnetic damper body, with $C_D x_D$ directed along the axis of the internal magnet towards its South Pole and arbitrarily directed two other axes that supplement frame of reference to the right rectangular one. This frame of reference may be congruent with $C_1 xyz$ at the initial instant.
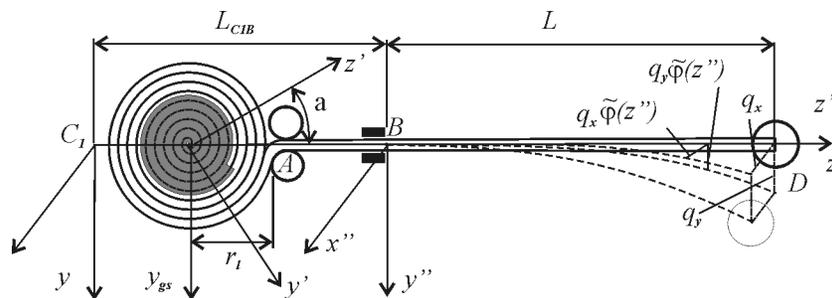
Fig. 3. Geometry of type

The external torques acting on the internal magnetic damper body consists of the torque of the geomagnetic field and torque of viscous friction. The second torque is caused either by liquid, which is filled in a gap between the magnetic damper central body and its case (liquid magnetic damper), or by the Foucault currents arising in a current-carrying case (induction magnetic damper) at relative rotation of the central magnet. Thus, motion of the magnetic damper central body should be described by a separate equation of motion. Further, without breaking a generality of approach, consideration will not be given beyond the liquid magnetic damper. In turn, the spacecraft is subjected to action of the torque from the gravitational field of the Earth, and the torque of a viscous friction from a liquid that surrounds the magnetic damper central body.

4. Mathematical Model of System

The equations of motion of the system under consideration become the most convenient and compact for numerical integration (both for attitude, and relative motions), if one chooses the instantaneous position of the mass center C as a pole. In this case, kinetic energy can be presented in the following form:

$$T_{S_1+S_2} = \frac{1}{2}\int_m v_i^2 dm = \frac{1}{2}\int_m (\vec{v}_C + \overset{*}{\vec{\rho}} + \vec{\omega}\times\vec{\rho})^2 dm =$$

$$\frac{1}{2}\int_m [\vec{v}_C^2 + \overset{*}{\vec{\rho}}{}^2 + (\vec{\omega}\times\vec{\rho})^2 + 2v_C\overset{*}{\vec{\rho}} + 2v_C(\vec{\omega}\times\vec{\rho}) + 2\overset{*}{\vec{\rho}}(\vec{\omega}\times\vec{\rho})]dm.$$

(1)

Here $\vec{\rho}$ is the position vector of an arbitrary point with respect to the instantaneous position of the mass center C.

After integration, the expression for the kinetic energy of the whole spacecraft is given by

$$T = \frac{M}{2}\vec{v}_C^2 + T_r^{C_1} - \frac{M}{2}\overset{*}{\vec{r}}{}'{}_c^2 + \frac{1}{2}\vec{\omega}\cdot\Theta^C\cdot\vec{\omega} + \vec{\omega}\cdot\vec{K}_r^C .$$

(2)

Here M is the total mass of the spacecraft, $\vec{v}_C$ is the absolute velocity of the mass center, and $T_r^{C_1}$ is the kinetic energy of the relative motion of the carried bodies, calculated under condition of definition of relative velocities of their points with respect to C_1. Note that though the kinetic energy of the relative motion is a scalar, it is necessary to specify a chosen pole for determination of the relative velocities for its correct calculation. It is easy to show that if to calculate the relative velocities of the specified points with respect to instantaneous position of the mass center C, the expression (2) is transformed to the most simple form:

$$T = \frac{M}{2}\vec{v}_C^2 + T_r^C + \frac{1}{2}\vec{\omega}\cdot\Theta^C\cdot\vec{\omega} + \vec{\omega}\cdot\vec{K}_r^C .$$

Now, as it is shown by Khoroshilov and Zakrzhevskii (2011), one can obtain the following Lagrange's equations of the second kind for the generalized co-ordinates $q_s (s = 1,2)$:

$$ \mathrm{E}_s(T_r^{C_1}) - M\, \overset{**}{\vec{r}_C}\, '\cdot \frac{\partial \vec{r}_C\,'}{\partial q_s} - \frac{1}{2} \vec{\omega} \cdot \frac{\partial \Theta^C}{\partial q_s} \cdot \vec{\omega} + \dot{\vec{\omega}} \cdot \frac{\partial \vec{K}_r^C}{\partial \dot{q}_s} + \vec{\omega} \cdot \mathrm{E}_s^*(\vec{K}_r^C) = Q_s . \tag{3} $$

The equation of the attitude motion may be obtained as the Euler-Lagrange equation

$$ \Theta^C \cdot \dot{\vec{\omega}} + \overset{*}{\Theta}^C \cdot \vec{\omega} + \vec{\omega} \times \left(\Theta^C \cdot \vec{\omega} \right) + \overset{*}{\vec{K}}_r^C + \vec{\omega} \times \vec{K}_r^C = \vec{m}^C . \tag{4} $$

Here Θ^C is the inertia tensor of the whole spacecraft with respect to point C; $\vec{K}_r^C$ is the relative moment of momentum of the deployed part with respect to point C; $\vec{\omega}$ is the vector of absolute angular velocity of the main body; $\vec{m}^C$ is the main torque of external forces with respect to the spacecraft mass center; $E_j(\cdot) = \dfrac{d}{dt} \dfrac{\partial (\cdot)}{\partial \dot{q}_j} - \dfrac{\partial (\cdot)}{\partial q_j}$ is Euler's operator; $E_j^*(\cdot) = \dfrac{\partial}{\partial t} \dfrac{\partial (\cdot)}{\partial \dot{q}_j} - \dfrac{\partial (\cdot)}{\partial q_j}$ is also Euler's operator, but time differentiation is done here in the reference frame $C_1 xyz$; $T_r^{C_1}$ is the kinetic energy of relative motion of the carried bodies in terms of their relative velocities in the frame of reference $C_1 xyz$; q_s is the generalized co-ordinates; Q_s is the generalized forces; $\dot{f}$ designates the absolute time derivative, and $\overset{*}{f}$ is the time derivative in the main body fixed reference frame. It is important to mention here that the term $\mathrm{E}_s^*(\vec{K}_r^C)$ does not come to equal $-2 \dfrac{\partial \vec{K}_r^C}{\partial q_s}$, as it is shown by Lurie (2002) since the case considered here is the more general case of non-stationary constraints.

For the description of the dynamics of the magnetic damper central body, it is necessary to use the equation

$$ \Theta^{C_d} \cdot \dot{\vec{\omega}}_d + \vec{\omega}_d \times \left(\Theta^{C_d} \cdot \vec{\omega}_d \right) = \vec{m}_d^{C_d} , \tag{5} $$

where Θ^{C_d} is the inertia tensor of the magnetic damper central body, $\vec{\omega}_d$ is the vector of absolute angular velocity of this body, $\vec{m}_d^{C_d}$ is the torque of the external forces. This torque consists of the torque arising due to magnetic damper and the torque of viscous forces, arising at rotation of the central magnetic damper body relative to its case.

Mass of fully deployed boom is essentially smaller of the boom tip mass, and the second natural frequency of such a boom is essentially greater than the first one. Therefore, only one mode of elastic oscillations in each of the main planes of the fixed in the spacecraft frame of reference can be taken into consideration. It is important to mention here that natural modes and frequencies of oscillations vary during the boom deployment. Because of a small spacecraft angular velocity and slow gravitational stabilizer deployment the effect of longitudinal forces on modes of oscillations may be neglected. With the use of Krylov's functions that are described by Lurie (2002) for representation of elastic Eigen-modes in two planes for a beam with one fixed end and point mass on the free end it is possible to show that the corresponding characteristic equation can be written as:

$$\mathrm{V}(kl)\,\mathrm{T}(kl) - \mathrm{S}(kl)^2 + kl\,\frac{m_{gs}}{m_b}(\mathrm{T}(kl)\;\mathrm{U}(kl) - \mathrm{V}(kl)\,\mathrm{S}(kl)) = 0, \tag{6}$$

where $\mathrm{S}(kl), \mathrm{T}(kl), \mathrm{U}(kl), \mathrm{V}(kl)$ are Krylov's functions, kl are roots of the characteristic equation, m_{gs} and m_b are the mass on the end and mass of the deployed part of the beam accordingly. The equation (6) is singular at the initial instant of deployment of the beam (at $m_b = 0$). This fact needs to be taken into consideration at the numerical simulation. Torsion oscillations of the tip mass relative to the longitudinal axis of the gravitational stabilizer may be neglected, as they do not change components of the spacecraft inertia tensor and cannot make an impact upon spacecraft angular oscillations. A somewhat different model of a console beam with the end mass placed on the mobile basis, but without taking into account mass of the beam, was considered by Genta (2013). Banerjee and Kane (1989) developed a method for mathematical description of the dynamics of a beam that is being extruded from, or retracted into, a rigid rotating body. The method consisted in modeling the beam as a series of elastically connected rigid links and then working with equations of motion linearized in the modal coordinates for the links outside the rigid body at a given time.

The drum with the reeled up tape can be considered as a handwheel with the variable moment of inertia, and body S_2 can be considered as a body with the mass center C_2 that is moved with respect to C_1 and having a variable inertia tensor.

Expressions for the various integrals concerned with the modes of elastic displacements, which are used for calculation of factors of the equations of motion, look like:

$$F_1(t) = \int_0^{s(t)} \tilde{\varphi}(\tilde{z},t)\,d\tilde{z}, \quad F_2(t) = \int_0^{s(t)} \tilde{\varphi}(\tilde{z},t)(\tilde{z} + L_{C_1 B})\,d\tilde{z},$$

$$F_3(t) = \int_0^{s(t)} (\tilde{\varphi}(\tilde{z},t))^2 d\tilde{z}, \quad F_4(t) = \int_0^{s(t)} \left(\frac{\partial^2 \tilde{\varphi}(\tilde{z},t)}{\partial \tilde{z}^2} \right)^2 d\tilde{z}. \tag{7}$$

Here $\tilde{\varphi}(\tilde{z},t)$ is the first normalized natural mode of transverse oscillations of an elastic beam lengthwise $s(t)$ with point mass m_{gs} on its tip. Its dependence on time is due to change of value m_{gs}/m_b in the characteristic equation (6) during the gravitational stabilizer deployment. As an example, it is possible to refer to Fig. 4 where one can see how ordinates of the first normalized natural mode change during the boom deployment.
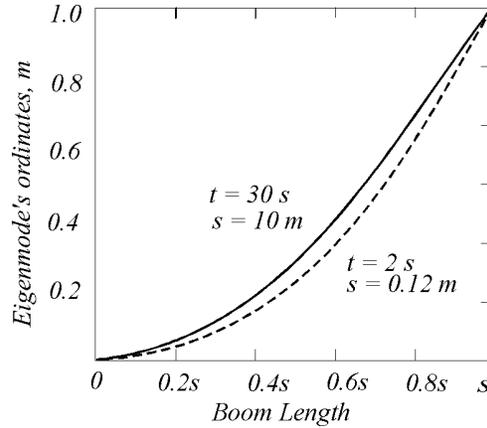

Fig. 4. The first Eigen-mode of cantilever beam with a tip mass versus time.

Basic dynamic values

All analytical expressions for coefficients of the dynamic equations are derived in Mathematica5©. Position vectors of the specific points of the system under consideration in the main body fixed frame of reference are: for the magnetic damper mass center $\vec{r}_D = \{q_x, q_y, s + L_{C_1B}\}$; for the mass center of the bent beam $\vec{r}_{C_l}{}' = \{q_x / L\, F_1(t), q_y / L\, F_1(t), L/2 + L_{C_1B}\}$; for the mass center of the tape on the drum $\vec{r}_O{}' = \{0,0,L_{C_1O}\}$. Current mass of the tape on the drum is $m_{dL} = m_L(L-s)$, where L is length of the completely deployed boom; external radius of the tape that has been reeled up on the drum is $r_L = r_0 + \kappa(\Phi - \psi)$. Here $\kappa = \delta / 2\pi$, δ is tape thickness; m_l is tape mass per unit length; Φ is the maximum value of the rotation angle of the drum during the gravitational stabilizer deployment; q_x, q_y are the generalized co-ordinates of flexible displacements determining a deflection of the boom end in planes $C_1 xz, C_1 yz$ accordingly.

Geometry of the type in the device of deployment is shown in Fig. 3. Here $C_1 xyz$ is the main body fixed frame of reference. One can treat the central line of the tape to be wound on the drum as the Archimedes right spiral $\rho = \dfrac{\delta}{2\pi}\varphi_k$, $\varphi_k \in [0, \Phi_f]$, where δ is tape thickness. The part of this spiral, for which $\varphi_k \in [0, \varphi_D]$, forms a body of the drum (in Fig. 3 this area is grey and the spiral is shown by a dotted line in it). The tape area prior to the beginning of the deployment corresponds to values $\varphi_k \in [\varphi_D, \Phi_f]$.

In Fig. 3, point A is the point where the tape leaves the drum. Point G is an arbitrary point of the spiral. Total length of the tape is wound on the drum at the initial instant and each point of its middle surface lies at the spiral. Point D is on the tape starting at the drum. It corresponds to the angular co-ordinate φ_D of the spiral. If r_0 is the distance from point O_1 to the drum center D, $\varphi_D = 2\pi r_0 / \delta$.

The length of the spiral on the drum is

$$L_D = \frac{\delta}{4\pi}[\varphi_D\sqrt{1+\varphi_D^2} + \ln(\varphi_D + \sqrt{1+\varphi_D^2})]. \tag{8}$$

At the initial time, point A is positioned at the spiral end. Value φ_A can be found numerically using known overall length of the tape, which forms the gravitational stabilizer, from the condition

$$L_{AD} = \frac{\delta}{4\pi}[\varphi_A\sqrt{1+\varphi_A^2} + \ln(\varphi_A + \sqrt{1+\varphi_A^2})] - \frac{\delta}{4\pi}[\varphi_D\sqrt{1+\varphi_D^2} + \ln(\varphi_D + \sqrt{1+\varphi_D^2})] = L, \tag{9}$$

where L is length of the deployed gravitational stabilizer. Here one must take into account the fact that point A is moving to the drum center for the distance $\delta\psi_d/(2\pi)$ as the tape is unwinding.

Total angle of winding of the tape on the drum $\Phi_L = \varphi_A - \varphi_D$. The length of the tape s forming the gravitational stabilizer at rotation of the drum at an arbitrary angle ψ_d is equal to:

$$\begin{aligned}
s &= \frac{\delta}{4\pi}[\varphi_A\sqrt{1+\varphi_A^2} + \ln(\varphi_A + \sqrt{1+\varphi_A^2})] \\
&- \frac{\delta}{4\pi}[(\varphi_A - \psi_d)\sqrt{1+(\varphi_A-\psi_d)^2} + \ln((\varphi_A - \psi_d) + \sqrt{1+(\varphi_A-\psi_d)^2})] - \delta\psi_d/(2\pi).
\end{aligned} \tag{10}$$

Double differentiation of the expression (10) with respect to time gives expressions for $\dot{s}$ and $\ddot{s}$, which are necessary to calculate coefficients of the dynamical equations. In differentiation of this expression it is necessary to consider that φ_A is a constant here, which is determined by an initial condition of the drum prior to the beginning of deployment.

As shown in Fig. 3, the components of the position vector of an arbitrary material point of the deployed gravitational stabilizer boom with regard to its elastic displacements in the frame of reference C_1xyz look like

$$\vec{r}'_L = \{q_x(t)\tilde{\varphi}(\tilde{z},t), q_y(t)\tilde{\varphi}(\tilde{z},t), C_1B + \tilde{z}\}. \tag{11}$$

The vector of a relative velocity of such a material point cannot be obtained from this expression by simple time differentiation in the frame of reference C_1xyz since any material point of the gravitational stabilizer boom executes an additional motion along an axis C_1z'' with the velocity $\dot{s}(t)$ during deployment. Therefore

$$\vec{v}'_L = \frac{\partial \vec{r}'_L}{\partial t} + \{0,0,\dot{s}(t)\}. \tag{12}$$

Taking into account that Eigen-mode $\tilde{\varphi}(\tilde{z},t)$ depends on time, expression for $\partial \vec{r}'_L / \partial t$ can be written as

$$\partial \vec{r}'_L / \partial t = \{\dot{q}_x(t)\tilde{\varphi}(\tilde{z},t) + q_x(t)\tilde{\varphi}^{(0,1)}(\tilde{z},t), \dot{q}_y(t)\tilde{\varphi}(\tilde{z},t) + q_y(t)\tilde{\varphi}^{(0,1)}(\tilde{z},t), 0\}. \tag{13}$$

For the point end mass, the same values can be determined as

$$\vec{r}'_G = \{q_x(t), q_y(t), C_1B + z''\}. \tag{14}$$

$$\vec{v}'_G = \frac{\partial \vec{r}'_G}{\partial t} + \{0,0,\dot{s}(t)\}. \tag{15}$$

The components of the spacecraft inertia tensor Θ^{C_1} in the reference frame C_1xyz can be written taking into account the expressions (7) as follows:

$$\Theta^{C_1}_{xx} = J_{xx} + m_{dl}(\frac{r_l^2 + r_0^2}{2} + L^2_{C_1O}) + m_{gs}(r_D(2)^2 + r_D(3)^2) + m_l(\frac{z_D^3 - z_B^3}{3} + q_y^2 F_3),$$

$$\Theta^{C_1}_{yy} = J_{yy} + m_{dl}(\frac{3r_l^2 + 3r_0^2 + b^2}{12} + L^2_{C_1O}) + m_{gs}((r_D(1)^2 + r_D(3)^2)) + m_l(\frac{z_D^3 - z_B^3}{3} + q_x^2 F_3),$$

$$\Theta^{C_1}_{zz} = J_{zz} + m_{dl}\frac{3r_l^2 + 3r_0^2 + b^2}{12} + m_{gs}(r_D(1)^2 + r_D(2)^2) + m_l(q_x^2 + q_y^2)F_3,$$

$$\Theta^{C_1}_{xy} = J_{xy} + (m_{gs} + m_l F_3)\, q_x q_y, \Theta^{C_1}_{xz} = J_{xz} + (m_{gs}(L_{C_1B} + L) + m_l F_2)\, q_x,$$

$$\Theta^{C_1}_{xy} = J_{xy} + (m_{gs} + m_l F_3)\, q_x q_y, \Theta^{C_1}_{xz} = J_{xz} + (m_{gs}(L_{C_1B} + L) + m_l F_2)\, q_x. \tag{16}$$

Here $J_{xx}, J_{xy,...}$ are components of the inertia tensor of the gyrostatic system part. The inertia tensor of the magnetic damper with respect to its mass center has negligible components in comparison with its contribution to the inertia tensor of the spacecraft as a whole. Components of the inertia tensor are calculated in the reference frame C_1xyz, therefore the following ratio is used further in the equations of motion: $\Theta^C = \Theta^{C_1} - M(E\vec{r}'_C \cdot \vec{r}'_C - \vec{r}'_C\vec{r}'_C)$. The mass of the magnetic damper central

body is taken into account in the calculation of the spacecraft inertia tensor, as the mass center of the magnetic damper is motionless with respect to its case, which is the spacecraft element.

The kinetic energy of gravitational stabilizer elements, if one calculates the relative velocities with respect to point C_1, is governed by

$$T_r^{C_1} = J_d \dot{\psi}^2 / 2 + m_{gs}(\dot{q}_x^2 + \dot{q}_y^2)\varphi^2(L)/2 + m_l[(L_{ab} + L)\dot{L}^2(\dot{q}_x^2 + \dot{q}_y^2)F_3] \qquad (17)$$

The moment of momentum of the gravitational stabilizer elements in the relative motion can be written as

$$\vec{K}_r^{C_1} = \{[J_{dr} + m_{dl}(r_0^2 + r_l^2)/2]\dot{\psi} + m_{gs}(q_y\dot{L} - \dot{q}_y(L_{C_1B} + L)) + m_l(-F_2\dot{q}_y + F_1q_y\dot{L})],$$
$$m_{gs}(q_x\dot{L} - \dot{q}_x(L_{C_1B} + L)) + m_l(-F_2\dot{q}_x + F_1q_x\dot{L})], (m_{gs} + m_lF_3(q_x\dot{q}_y - \dot{q}_xq_y)\} . \qquad (18)$$

The moment of momentum of the relative rotation of the magnetic damper central body does not enter into this expression, as its combination with the spacecraft is purely dynamic. Interaction of the spacecraft and magnetic damper implements only through the torque of a viscous friction with rotation of the central body in the magnetic damper case, which can be considered as the external torque both for the spacecraft and for magnetic damper central body.

Potential energy of forces of elasticity for the system under consideration can be written as follows:

$$\Pi_e = (\frac{q_x^2}{2} + \frac{q_y^2}{2}) \int_0^{L(t)} EJ(\frac{\partial^2\varphi}{\partial(z'')^2})^2 dz'' = (\frac{q_x^2}{2} + \frac{q_y^2}{2}) F_4(t). \qquad (19)$$

5. External Torques and Generalized Forces

The torque $\vec{m}^C$ in Eq. (4) consists of the gravitational torque arising due to interaction of the spacecraft with the gravitational field of the Earth, and the torque of forces of a viscous friction due to rotation of the magnetic damper central body with respect to its case.

The inclined dipole is considered as a model of the magnetic field of the Earth. Its axis is determined by the ort $\vec{e}_m$, which is turned down from rotation axis of the Earth on 11.5o and originates from the center of the Earth (Fig. 5). The longitude of the northern magnetic pole at the initial instant may be determined by Julian date of beginning of the numerical simulation.
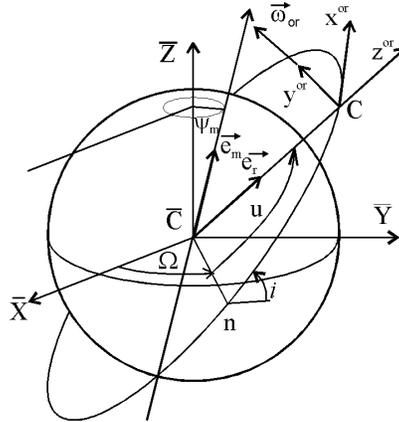
Fig. 5. Inclined dipole and orbital reference frame.

In the accepted absolute frame of reference.

$$\vec{e}_m = \{\sin\alpha_m \cdot \cos\psi_m, \sin\alpha_m \cdot \sin\psi_m, \cos\alpha_m\}, \tag{20}$$

$$\vec{e}_r = \{\cos\Omega \cdot \cos u - \sin\Omega \cdot \sin u \cdot \cos i, \\ \sin\Omega \cdot \cos u + \cos\Omega \cdot \sin u \cdot \cos i, \sin u \cdot \sin i\}. \tag{21}$$

Going after Beletsky and Levin (1993), the vector of the magnetic induction at any point of an orbit may be written in the form:

$$\vec{B} = \mu_m R^{-3}[\vec{e}_m - 3(\vec{e}_m \cdot \vec{e}_r) \cdot \vec{e}_r]. \tag{22}$$

The algorithm of calculation of the torques and generalized forces concerned with the magnetic damper consists of the following. The torque $\vec{m}_d^{C_d}$ in the equation (5) is determined by interaction of the constant magnet of the magnetic damper central body with the magnetic field of the Earth. It can be written by

$$\vec{m}_d^{C_d} = \vec{m}_{md} \times \vec{B}, \tag{23}$$

where $\vec{m}_{md}$ is the vector of the magnetic torque of the constant magnet.

With help of the relations determining mutual transformation of frames of reference (see below Eq. (32)), it is possible to calculate projections of the vector of the magnetic torque on the orbital frame of reference. Calculating projections of the vectors (20), (21) and (22) on the orbital frame of reference, it is possible to calculate $\vec{m}_d^{C_d}$ with the formula (23) in the orbital frame of reference also and to transform it to the frame of reference $C_D x_D y_D z_D$, then to substitute the obtained expression in Eq. (5) and find projections of the vectors of absolute angular velocities of the spacecraft and magnetic damper on their frames of reference. Further, it is necessary to find the projections of these vectors on the orbital frame of reference. After finding of the angular-velocity vector of the

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magnetic damper central body with respect to its case under the formula (25), the torque of a viscous friction that acts on the magnetic damper case can be calculated in the orbital frame of reference. Projections of this torque on axes $C_1 xyz$ can be used further for calculation of the generalized forces. Its projections on the frame of reference $C_D x_D y_D z_D$ with the inverse sign can be used in Eq. (5) as the additional external torque from the magnetic field of the Earth.

In view of Eq. (19), the expressions for the generalized forces of elastic displacements can be written as

$$Q_x = -EJF_4 q_x, Q_y = -EJF_4 q_y. \qquad (24)$$

It is necessary to add to them the generalized forces from action of the damping torque of forces of a viscous friction on the magnetic damper case and from the structural damping. The first torque can be presented as

$$\vec{M}_{visc} = k_d \vec{\omega}_{rel}. \qquad (25)$$

The corresponding virtual work is given by

$$\delta A_{visc} = M_{visc}(2)\delta\gamma_x - M_{visc}(1)\delta\gamma_y.$$

Here γ_x, γ_y are the angles of rotation of the boom end in the point of attachment of the magnetic damper case (Fig. 6) in planes $C_1 xz, C_1 yz$ accordingly. The gravitational stabilizer boom is considered as structure that is rigid with respect to torsion, therefore the virtual work of the component $M_{visc}(3)$ vanishes. Since

$$\gamma_x = Arctg(q_x \frac{\partial \tilde{\varphi}(z'')}{\partial z''}\big|_{z''=L}), \ \gamma_y = -Arctg(q_y \frac{\partial \tilde{\varphi}(z'')}{\partial z''}\big|_{z''=L}), \qquad (26)$$

(the sign "–" in the second formula is determined by the fact that at positive value q_y the rotation on the angle γ_y occurs clockwise if to look from the end of axes $C_1 x$) their variations look as follows:
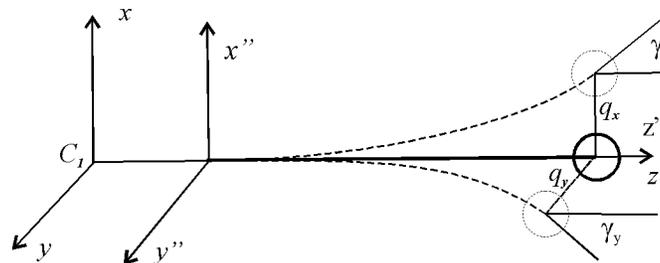

Fig. 6. Flexible deformation of gravitational stabilizer boom.

$$\delta\gamma_x = \frac{\partial\tilde{\varphi}(z'')}{\partial z''}\Big|_{z''=L}/(1+q_x^2(\frac{\partial\tilde{\varphi}(z'')}{\partial z''}\Big|_{z''=L})^2)\delta q_x, \quad \delta\gamma_y = -\frac{\partial\tilde{\varphi}(z'')}{\partial z''}\Big|_{z''=L}/(1+q_y^2(\frac{\partial\tilde{\varphi}(z'')}{\partial z''}\Big|_{z''=L})^2)\delta q_y,$$

whence it follows that

$$Q_{MDx} = M_{visc}(2)\frac{\partial\tilde{\varphi}(z'')}{\partial z''}\Big|_{z''=L}/(1+q_x^2(\frac{\partial\tilde{\varphi}(z'')}{\partial z''}\Big|_{z''=L})^2),$$

$$Q_{MDy} = -M_{visc}(1)\frac{\partial\tilde{\varphi}(z'')}{\partial z''}\Big|_{z''=L}/(1+q_y^2(\frac{\partial\tilde{\varphi}(z'')}{\partial z''}\Big|_{z''=L})^2) \tag{27}$$

Here $k_d = 8/3\pi\rho r_0^4/\varepsilon$ is the damping factor (Sokolov (1973)); ρ is viscosity of a liquid in the damper; r_0 is radius of the case; ε is the gap between the case and magnetic damper central body, $M_{visc\,x}$, $M_{visc\,y}$ are projections of the vector $\vec{M}_{visc}$ on the reference frame C_1xyz.

The magnetic damper case not only moves in the reference frame C_1xyz at the elastic deformations of the gravitational stabilizer boom, it also rotates in two planes (see Fig. 6). These rotations should be taken into account at the determination of the vector of angular velocity $\vec{\omega}_{rel}$ of the magnetic damper central body with respect to its case. Their contribution in $\vec{\omega}_{rel}$ can be essential at considerable oscillations of the gravitational stabilizer boom. Therefore, it is expedient to define the absolute angular velocity of the magnetic damper case in the reference frame C_1xyz as $\vec{\omega}_{Corp} = \vec{\omega}_{KA} + \dot{\gamma}_y\vec{i}_x + \dot{\gamma}_x\vec{i}_y$. Taking into consideration (26), one can write

$$\vec{\omega}_{rel} = \vec{\omega}_{MD} - \vec{\omega}_{KA} + \frac{\partial\tilde{\varphi}(z'')}{\partial z''}\Big|_{z''=L}/(1+q_y^2(\frac{\partial\tilde{\varphi}(z'')}{\partial z''}\Big|_{z''=L})^2)\dot{q}_y\vec{i}_x$$

$$-\frac{\partial\tilde{\varphi}(z'')}{\partial z''}\Big|_{z''=L}/(1+q_x^2(\frac{\partial\tilde{\varphi}(z'')}{\partial z''}\Big|_{z''=L})^2)\dot{q}_x\vec{i}_y \tag{28}$$

Projections of $\vec{M}_{visc}$ on C_1xyz will define also the additional external torque acting on the spacecraft.

The generalized forces of a structural damping can be introduced formally as $Q_{x,y} = -k_{1x,y}\dot{q}_{x,y}$ reasoning from a condition that decrement of oscillations of such a system as a cantilever metal beam with mass on the tip should have the value about 0.01.

6. Kinematic Equations of System

It is expedient to use the Rodriguez-Hamilton parameters for determination of orientation both the main spacecraft module and internal magnetic damper body. The kinematic equations for the spacecraft can be written in the scalar form as follows:

$$2\dot{\lambda}_0 = -\tilde{\omega}_1\lambda_1 - \tilde{\omega}_2\lambda_2 - \tilde{\omega}_3\lambda_3, \quad 2\dot{\lambda}_1 = \tilde{\omega}_1\lambda_0 + \tilde{\omega}_3\lambda_2 - \tilde{\omega}_2\lambda_3,$$

$$2\dot{\lambda}_2 = \tilde{\omega}_2\lambda_0 + \tilde{\omega}_1\lambda_3 - \tilde{\omega}_3\lambda_1, \quad 2\dot{\lambda}_3 = \tilde{\omega}_3\lambda_0 + \tilde{\omega}_2\lambda_1 - \tilde{\omega}_1\lambda_2.$$

(29)

Here $\lambda_0, \lambda_1, \lambda_2, \lambda_3$ are the quaternion components that transform the orbit reference frame to the one fixed in the main body; $\tilde{\omega}_i = \omega_i - \omega'_{oi}$ $(i = 1,2,3)$; ω_i are the projections of the vector of the absolute angular velocity of the main body on the axes of the main body reference frame; ω'_{oi} are the projections of the vector of the orbital angular velocity on the same frame. As the orbital angular velocity is collinear with axis Cy^{or}, its projections on the axes of the reference frame fixed in the main body are defined by the matrix of the corresponding direction cosines written in the quaternion components (Lurie (2002)). As a result:

$$\omega'_{o1} = 2(\lambda_0\lambda_3 + \lambda_1\lambda_2)\omega_o, \quad \omega'_{o2} = (\lambda_0^2 + \lambda_2^2 - \lambda_1^2 - \lambda_3^2)\omega_o, \quad \omega'_{o3} = 2(-\lambda_0\lambda_1 + \lambda_2\lambda_3)\omega_o. \quad (30)$$

The matrix T_{OrbSv} of transition from the orbital reference frame to one fixed in the spacecraft in the Rodriguez-Hamilton parameters (Branets and Shmyglevsky, 1973; Lurie, 2002) looks like

$$
\begin{array}{c}
\quad x_{orb} \qquad\qquad y_{orb} \qquad\qquad z_{orb} \\
\begin{array}{c} x_{sv} \\ y_{sv} \\ z_{sv} \end{array}
\begin{vmatrix}
\lambda_0^2 + \lambda_1^2 - \lambda_2^2 - \lambda_3^2 & 2(\lambda_0\lambda_3 + \lambda_1\lambda_2) & 2(-\lambda_0\lambda_2 + \lambda_1\lambda_3) \\
2(-\lambda_0\lambda_3 + \lambda_1\lambda_2) & \lambda_0^2 - \lambda_1^2 + \lambda_2^2 - \lambda_3^2 & 2(\lambda_0\lambda_1 + \lambda_2\lambda_3) \\
2(\lambda_0\lambda_2 + \lambda_1\lambda_3) & 2(-\lambda_0\lambda_1 + \lambda_2\lambda_3) & \lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2
\end{vmatrix}.
\end{array}
$$

(31)

Now one can write

$$\vec{R}_{sv} = T_{OrbSv}\,\vec{R}_{Orb} \text{ or } \vec{R}_{Orb} = T_{OrbSv}^T\,\vec{R}_{Sv} = T_{SvOrb}\,\vec{R}_{Sv}.$$

(32)

The kinematic equations for the central magnetic damper body can be written in the scalar form as follows:

$$2\dot{\lambda}_0^D = -\tilde{\omega}_1^D\lambda_1^D - \tilde{\omega}_2^D\lambda_2^D - \tilde{\omega}_3^D\lambda_3^D, \quad 2\dot{\lambda}_1^D = \tilde{\omega}_1^D\lambda_0^D + \tilde{\omega}_3^D\lambda_2^D - \tilde{\omega}_2^D\lambda_3^D,$$

$$2\dot{\lambda}_2^D = \tilde{\omega}_2^D\lambda_0^D + \tilde{\omega}_1^D\lambda_3^D - \tilde{\omega}_3^D\lambda_1^D, \quad 2\dot{\lambda}_3^D = \tilde{\omega}_3^D\lambda_0^D + \tilde{\omega}_2^D\lambda_1^D - \tilde{\omega}_1^D\lambda_2^D.$$

(33)

Here $\lambda_0^D, \lambda_1^D, \lambda_2^D, \lambda_3^D$ are the quaternion components that transform the orbit reference frame to the one fixed in the magnetic damper; $\tilde{\omega}_i^D = \omega_i^D - \omega'^D_{oi}$ $(i = 1,2,3)$; ω_i^D are the projections of the vector of the absolute angular velocity of the central magnetic damper body on axes of the reference frame fixed in it; ω'^D_{oi} are the projections of the vector of the orbital angular velocity on

the same axes. The projections of the orbital angular velocity on axes fixed in the magnetic damper are defined by analogy with (30):

$$\omega'^D_{o1} = 2(\lambda^D_0 \lambda^D_3 + \lambda^D_1 \lambda^D_2)\omega_o, \; \omega'^D_{o2} = ((\lambda^D_0)^2 + (\lambda^D_2)^2 - (\lambda^D_1)^2 - (\lambda^D_3)^2)\omega_o,$$

$$\omega'^D_{o3} = 2(-\lambda^D_0 \lambda^D_1 + \lambda^D_2 \lambda^D_3)\omega_o. \tag{34}$$

The proper matrix of transition from the orbital reference frame to the reference frame fixed in the central magnetic damper body can be constructed by the analogy with (31) by replacing the parameters $\lambda_0, \lambda_1, \lambda_2, \lambda_3$ in it with the parameters $\lambda^D_0, \lambda^D_1, \lambda^D_2, \lambda^D_3$. As the Krylov's angles that are more common used for the illustration of the spacecraft attitude motion it is possible to introduce the sequence of rotations shown in Fig. 7. Here φ is the roll angle, ϑ is the pitch angle, and ψ is the yaw angle.
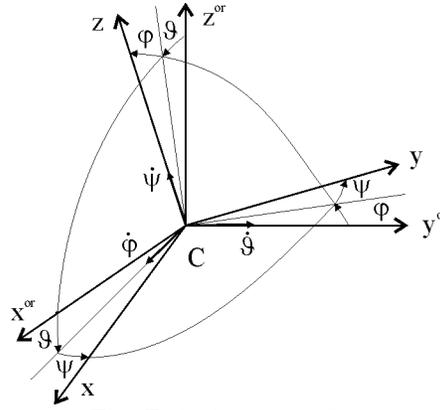
Fig. 7. Attitude angles.

7. Simulations

The following values of the base parameters were used for the numerical simulation. The spacecraft circular orbit has the radius 6700 km with the inclination $83°$. Mass of the spacecraft main body $m_1 = 550$ kg; mass of the magnetic damper $m_{MD} = 10$ kg; own magnetic moment of the magnetic damper $m_{MD} = 500$ J/Tesla; the magnetic damper damping factor $k_d = 1.4$ N m s; the bending stiffness of the gravitational stabilizer boom in the deployed position $EJ = 200$ N m²; the deployed length of the gravitational stabilizer boom L = 10 m; the tape mass per unit length $m_L = 0.17$ kg/m; the diagonal components of the inertia tensor of the spacecraft main module in the reference frame $C_1 xyz$: $\Theta^{C_1}_{11} = 4000$ kg m², $\Theta^{C_1}_{22} = 6000$ kg m², $\Theta^{C_1}_{33} = 2000$ kg m²; the moment of momentum of the pitch handwheel $H_{FW} = 10$ kg m²/s; the duration of the deployment T$_f$-T$_0$ = 100-400 s; the deployment law corresponds to a smooth attainment of constant velocity and the same completion of the process that is optimal in the sense of minimization of relative accelerations of elastic displacements (Zakrzhevskii (2008)). The numerical integration of the initial value problem is done

by the Runge-Kutta method of fourth order with the variable step of integration within the range 0.01-0.001s. The numerical simulation begins with the instant of time when initial damping of the spacecraft attitude motion in the orbital frame of reference is finished.

In the beginning it is expedient to consider behavior of the magnetic damper in the magnetic field of the Earth with the parameters resulted above, located on ideally stabilized platform, in the Earth orbits having various inclinations. It will allow developing some insight into possibilities of such a damping system.

In Fig. 8, 9 the time histories of the orientation angles of the central magnetic damper body and components of the torque acting on this body from the magnetic field of the Earth in the orbital reference frame, for orbits with inclination $45°$, $83°$, $90°$ and for the orbit laying at the initial moment in the plane of the magnetic equator (ME), marked by corresponding designations, are represented. In the hypothetical orbit laying in the plane of the magnetic equator of the uninclined dipole the vector of the own magnetic damper magnetic moment, which is directed in the initial moment along C_1x, rotates on the angle $\pi/2$ around the axis C_1z and further supports such orientation. It is obvious that thus the magnetic damper can damp both spacecraft oscillations with regard to axes C_1x and C_1z, and gravitational stabilizer elastic oscillations in plane C_1yz. The magnetic damper will not interact with other possible oscillations. The situation close to similar can be assumed also in the real equatorial orbit taking into account inclination of the magnetic dipole. In a real orbit with inclination $45°$, the magnetic damper rotates on all three attitude angles. Since its rotation is not synchronized with the orbital reference frame, some perturbations will be brought in the spacecraft orientation, though they can be insignificant at low requirements to orientation accuracy. At the same time large perturbations of the attitude oscillations and elastic oscillations concerned with them can be damped. On near-polar orbits with inclination $83°$ and $90°$ the basic rotation of the magnetic damper central body occurs around an axis that is disposed nearly the pitch axis, and during one spacecraft rotation around the Earth the magnetic damper rotates in the absolute frame of reference on the angle 4π. At that, there is the perturbing torque (Fig. 9), having an essential constant component. It tends to reject the spacecraft on a pitch angle from the local vertical. Naturally, the gravitational torque will counteract this perturbation, but the equilibrium position cannot correspond to the local vertical. Besides, from Fig. 9 it is visible that the greatest component of the torque acting on the magnetic damper magnet has an appreciable periodic component that increases at the equator and decreases near the poles. This component promotes occurrence of periodic errors of orientation. It is necessary to notice that the law of change of the magnetic field torque shown in Fig. 9 differs on small value from the viscous friction torque acting on the magnetic damper case coming from a viscous liquid since the inertial component, on which these two torques differ, is small in comparison with the torque due to viscous friction.
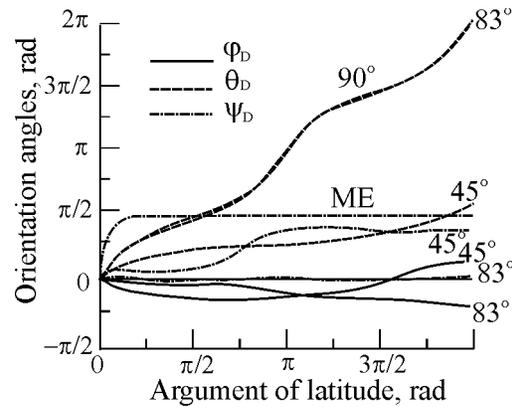
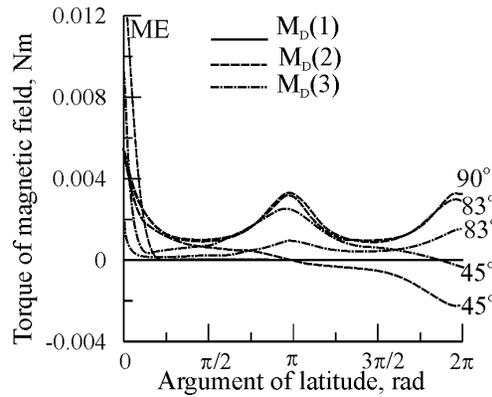**Fig. 8.** Time histories of orientation angles for orbits with various inclination.

Fig. 9. Components of magnetic torque acting on magnetic damper for orbits with various inclinations.

Simulation of the dynamics of the real spacecraft has allowed revealing a number of qualitative and quantitative features of the process of the passive spacecraft stabilization with respect to the local vertical.

In Fig. 10 the time histories of the attitude angles of the spacecraft without the magnetic damper and with it are shown after deployment of the flexible gravitational stabilizer. The spacecraft without magnetic damper that is stabilized at the instant of the beginning of the gravitational stabilizer deployment in the orbital frame of reference starts to rotate about the pitch axis with the negative relative angular velocity because of increase of the inertia tensor components Θ_{11}^C, Θ_{22}^C. As a result, the pitch angle, designated in Fig. 10 as ϑ_*, becomes negative. Under the effect of the restoring gravitational torque and in the absence of any damping, the spacecraft enters into a mode of harmonic oscillations around the pitch axis. Oscillations of other attitude angles in such an ideal case do not arise.
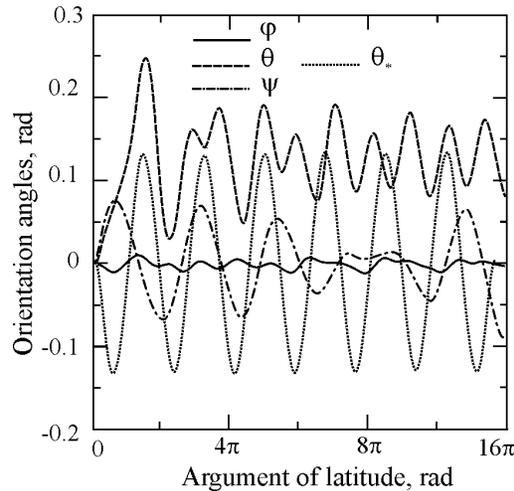
Fig. 10. Time histories of attitude angles for spacecraft without and with magnetic damper after deployment of elastic gravitational stabilizer.

Setting the magnetic damper on the spacecraft leads to the impossibility of passive orientation of the spacecraft reference frame strictly in the orbital reference one. It is caused by presence of the essential constant component in the torque of forces of a viscous friction that acts upon the spacecraft from the magnetic damper case, as it has been shown above. As a result of it even if to assume that in the beginning of the deployment the spacecraft has been stabilized in orbital frame of reference, the pitch angle ϑ will not get the negative area. The constant component of the moment of forces of a viscous friction on near-polar orbits will lead to inclination of the spacecraft around the pitch axis in the positive direction. The increase of the components $\Theta_{11}^C, \Theta_{22}^C$ of the inertia tensor will bring the impulse of the torque acting on all three axes of the spacecraft orientation. As a result, the oscillations arise around the pitch axis. Their amplitude will be limited by both the gravitational torque and the magnetic damper torque. Value of the pitch angle starts to fluctuate about some positive value with the gradually decreasing amplitude. Nevertheless, these oscillations cannot be damped, despite the dissipation of their energy by the magnetic damper. The non-uniform magnetic damper rotation around its axes prevents it. Through some rotations around the Earth, the amplitude of oscillations becomes practically constant. Average value of the pitch angle in Fig. 10 is equal approximately 0.12 rad. Naturally, axes of devices that should be parallel to the local vertical may be inclined in the spacecraft fixed frame of reference on the same angle, but in the opposite direction. In this case, the orientation error of the pitch angle will be defined by amplitude of the steady oscillations. From Fig. 10 it is visible that except the pitch oscillations at the gravitational stabilizer deployment and further interaction the magnetic damper with the magnetic field of the Earth essential oscillations of variable amplitude will arise also on yaw and roll angles. They will be especially essential for the yaw angle. These oscillations can turn into rotation with variable angular velocity at some combinations of parameters that can appear unacceptable for spacecraft working conditions.

A successful way for prevention of such a situation is setting on the spacecraft a handwheel with the constant vector of the moment of momentum that is parallel to the pitch axis. In Fig. 11 it is seen, how amplitudes of oscillations of the roll and jaw angles decrease thus.
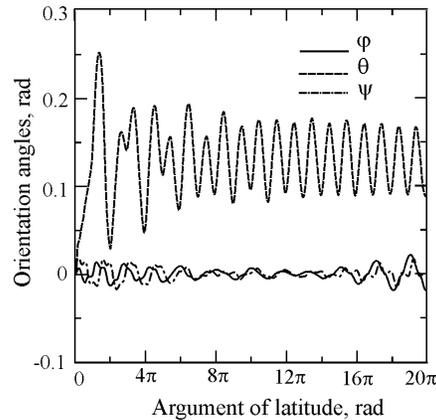
Fig. 11. Time histories of attitude angles for spacecraft with the magnetic damper and a pitch handwheel.

It is necessary to notice that a constant component of the pitch angle and the amplitude of oscillations essentially depend on the value of the magnetic damper magnetic moment and damping factor. In Fig. 12 it is seen that reduction of the damping factor three times in comparison with the previous case reduces both the average value, and amplitude of oscillations around the pitch axis. It naturally has an effect upon the amplitudes of oscillations around two other axes. Researches in this area have shown that for minimization of the orientation errors magnetic damper parameters should be selected for each orbit depending on its height.

In Fig. 13 the time histories of components of the vector of the spacecraft angular velocity concerning orbital frame of reference are resulted for a case of varying of the attitude angles that is shown in Fig. 11. Non-stationarity of these oscillations is caused by essential heterogeneity of the magnetic field for such orbits.

The dynamics of the flexible gravitational stabilizer and its interaction with the magnetic damper are worthy of studying separately. In Fig. 14, the time history of the generalized co-ordinates of gravitational stabilizer elastic displacements is shown in the case when the gravitational stabilizer is deployed from the preliminary stabilized spacecraft without the magnetic damper, using the optimal law of the deployment described above. The amplitude of an initial deviation of the generalized co-ordinate q_x depends on the bending stiffness of the gravitational stabilizer boom and on the duration of the deployment process. In the graph obtained, changes of the specified co-ordinate are insignificant, and the changes of the co-ordinate q_y are absent in general. In Fig. 15 that corresponds to the same spacecraft but with the magnetic damper on the gravitational stabilizer tip, the picture differs essentially. The internal magnetic damper body that rotates in its case located on the tip of the gravitational stabilizer elastic boom adds to the inertial influences of the spacecraft rotation the torque of a viscous friction. The vector of this torque does not coincide with the orbit plane. Therefore, there are deviations not only for co-ordinate q_x, but also for co-ordinate q_y.
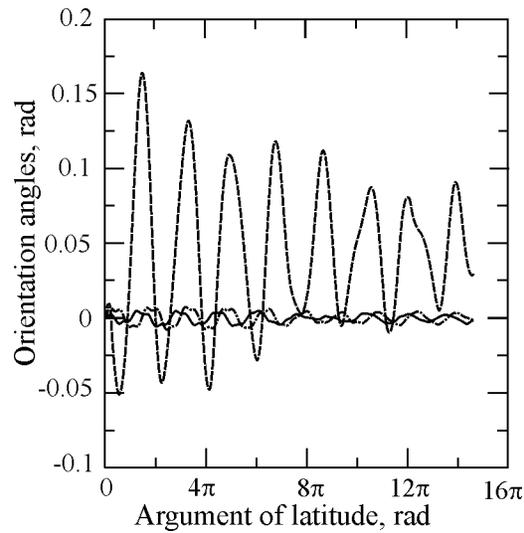
Fig. 12. Time histories of attitude angles for spacecraft with magnetic damper and pitch handwheel with reduction of damping factor.
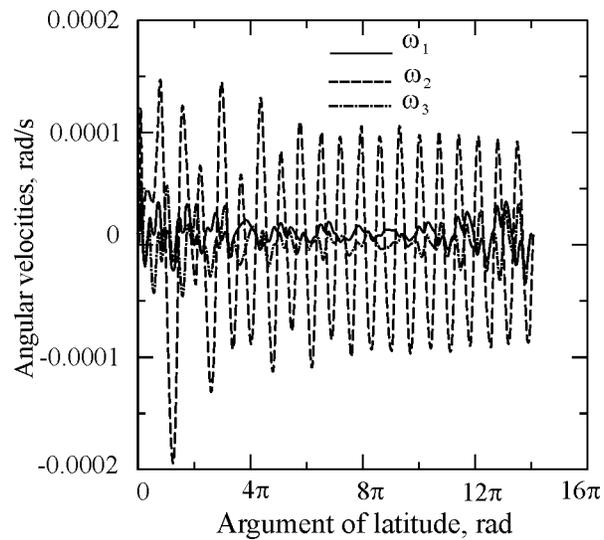
Fig. 13. Time histories of components of spacecraft angular velocity.
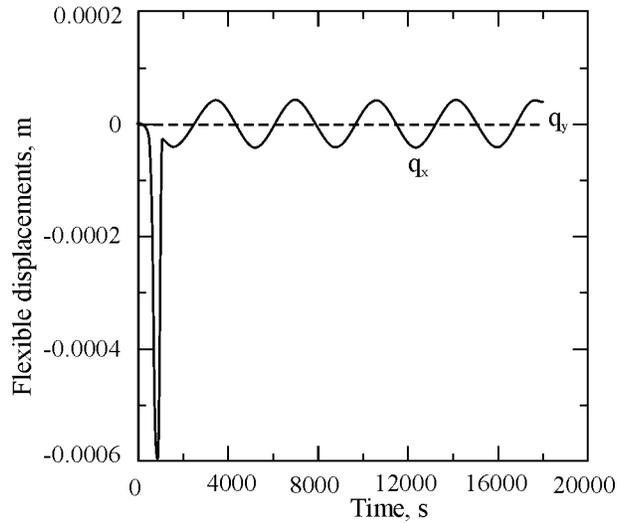
Fig. 14. Time history of generalized co-ordinates q_x, q_y in case of gravitational stabilizer deployment without magnetic damper.

Their non-stationarity is due to heterogeneity of the magnetic field along the spacecraft orbit. Oscillatory character of change of the generalized co-ordinates in these Figures is not due to natural oscillations of the flexible gravitational stabilizer boom. They are in one case (Fig. 14) result of oscillations of the spacecraft with the attached element in a gravitational field of forces, in other case (Fig. 15) perturbations from rotation of the magnetic damper central body in its case are imposed on such oscillations.
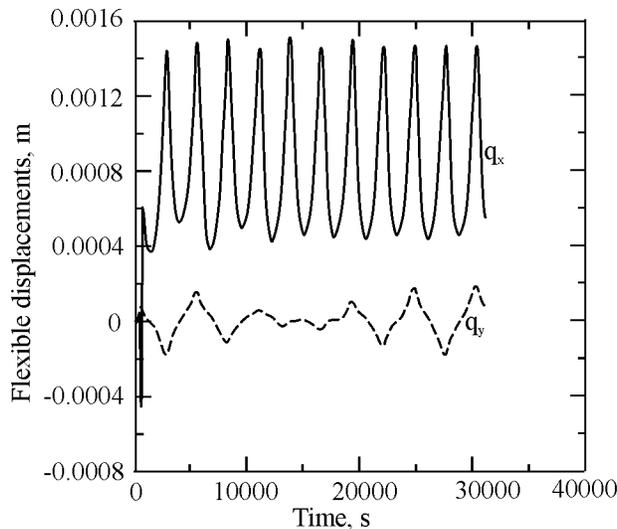

Fig. 15. Time history of generalized co-ordinates q_x, q_y in case of gravitational stabilizer deployment on spacecraft with magnetic damper.
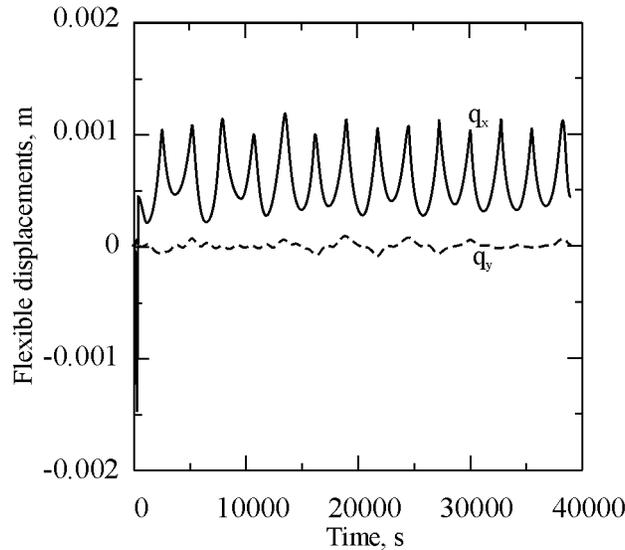
Fig. 16. Time history of generalized co-ordinates q_x, q_y in case of gravitational stabilizer deployment from spacecraft with magnetic damper with reduction of damping factor.

It is necessary to notice also that the variable component of the torque of a viscous friction somewhat promotes damping of the natural oscillations of the gravitational stabilizer boom.
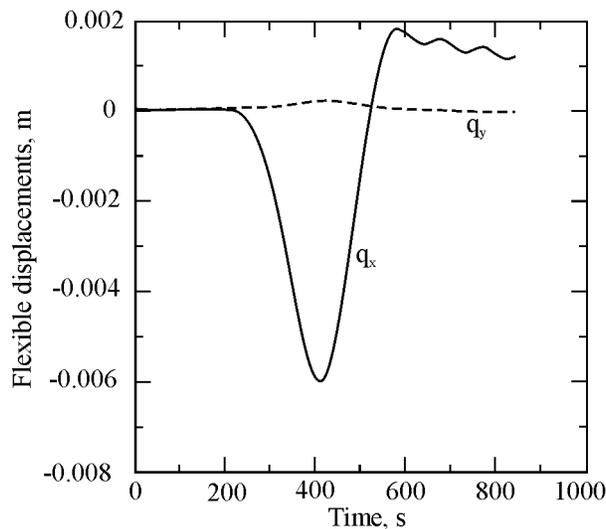

Fig. 17. Time histories of generalized co-ordinates q_x, q_y during gravitational stabilizer deployment.

With decrease of the magnetic damper effect on the elastic boom that occurs at decrease of the damping factor or at decrease of the value of the own magnetic moment of the magnetic damper central body, amplitudes of deviations for both generalized co-ordinates decrease. It is seen in Fig. 16. In Fig. 17 we show time histories of generalized co-ordinates q_x, q_y during the gravitational

stabilizer deployment ($T_0 = 200s$, $T_f = 600s$). Presence of natural oscillations of the boom is appreciable for co-ordinate q_x. They damp quickly enough under the effect of the structural damping. It is seen in all graphs that average value of gravitational stabilizer deviations in plane C_1xz is distinct from zero. The deviation value is defined by value of a constant component of the torque of a viscous friction in the magnetic damper. However, the carried out researches have shown that the structural damping plays main role in this process.

8. Conclusion

The present study deals with a study of the dynamics of the gravitationally stabilized spacecraft, which deploys the elastic gravitational stabilizer with energy dissipation of oscillations by the magnetic damper. The novel mathematical model, and numerical simulations are presented. Novelty of the approach consists in the taking into account additional internal degrees of freedom of the spacecraft in comparison with known earlier settings of the problem; in using the method developed of derivation of the dynamic equations of mechanical systems with internal degrees of freedom and non-stationary connections; in using of optimality with respect to damping of elastic oscillations control profile for deployment of flexible designs. The mathematical model developed for this case can be considered as generalization of the theory of flexible multi-body system with time dependent configurations. The approach can be successfully expanded on the simulation of the dynamics of other developed space structures with essential change of their configurations in the mode of their functioning. A FORTRAN package is created for the numerical simulation. It has general features that can be used easily for other deployed systems. The data obtained illustrate a method of the mathematical formulation of the dynamics of the system under consideration with program change of its geometry, as well as behavior of the spacecraft during and after deployment of the flexible structure. The information, which has been obtained with the numerical simulation, has led to a conclusion that deployment and the further functioning of the studied design can be acceptable for practice at a rational choice of parameters of the spacecraft design. The data obtained allow choosing the most suitable law of deployment and parameters of the magnetic damper. A detailed simulation study has allowed analyzing the dynamic behavior of the design at various values of parameters of the spacecraft with flexible gravitational stabilizer structure. Data obtained permit to the designer to select the most appropriate deployment, structure and damper parameters.

Acknowledgement

References

Alper, J. R., O'Neill, J. P.(1967). A new passive hysteresis damping technique for stabilizing gravity-oriented satellites. J. of Spacecraft and Rockets, 4(12), 1617-1622.

http://dx.doi.org/10.2514/3.29143

Beletsky V. V., Levin V. M. (1993). Dynamics of space tether systems, San Diego, Calif,: Univelt.

Banerjee, A. K., and Kane, T. R., (1989). Extrusion of a Beam from a Rotating Base. Journal of Guidance, Control, and Dynamics, 12(2), 140-146.
http://dx.doi.org/10.2514/3.20383

Branets, V. N., Shmyglevsky, I. P. (1973). Use of quaternions in the problems of rigid bodies orientation, Moscow: Nauka.

Crespoda Silva Marcelo, R. M. (1972). Non-linear resonant attitude – motion in gravity – stabilized gyrostat satellites. International Journal of Non – linear. Mechanics, 7(6), 621-641.
http://dx.doi.org/10.1016/0020-7462(72)90003-0

Genta, G. (2013). On the dynamics of a massless beam with end mass rotating in the three-dimensional space. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 227(3), 434-448
http://dx.doi.org/10.1177/0954406212463870

Khoroshilov, V. S., and Zakrzhevskii, A. E. (2011). Dynamics of spacecraft with gyro-gravitational system of stabilization due to elastic ring antenna deployment, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 225(10), 2333-2346
http://dx.doi.org/10.1177/0954406211410010

Lurie, A. I. (2002). Analytical mechanics. Springer,
http://dx.doi.org/10.1007/978-3-540-45677-3

Newton J. K., Farrell J. L. (1968). Natural frequencies of a flexible gravity – gradient satellite. Journal of Spacecraft and Rockets, 5(5), 560-569.
http://dx.doi.org/10.2514/3.29305

Sadov, Yu. A. (1969). Periodic motions of the spacecraft with magnetic damper in the plane of circular orbit, Kosmicheskie issledovanija, 7(1), 41-60.

Sokolov, L. V. (1973). Magnetic damper for system of gravitational orientation," In Control in Space, Proceedings of IV Int. Symp. on automatic contr. in space, Moscow: Nauka, (Vol. 1, pp.174-179).

Zakrzhevskii, A. E. (2013). Spacecraft dynamics with regard to elastic pantograph deployment, Journal of Spacecraft and Rockets. 50(2), 475-480
http://dx.doi.org/10.2514/1.A32397

Zakrzhevskii, A. E. (2008). Slewing of flexible spacecraft with minimal relative flexible acceleration, Journal of Spacecraft and Rockets. 31(3), 563–570