Vibration Suppression along the Segment of Beams by Imposing Nodes using Multiple Vibrations Absorbers

Sushil. S. Patil* and Pradeep. J. Awasare

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Abstract

Vibration absorbers are frequently used to suppress the excess vibrations in structural system. In this paper, imposing nodes technique is applied for vibration suppression of segment of Euler-Bernoulli beam subjected to force harmonic excitations using multiple vibration absorbers. The algorithm based on iterative procedure is developed that can be used to determine the required absorber parameters to impose nodes at selected locations on the beam. Numerical tests are performed to show the effectiveness of the proposed procedure. Experimental test is conducted on cantilever beam to verify the numerical results.

Keywords: Imposing nodes; Harmonic excitations; Vibration suppression; Cantilever beam; Absorbers

1. Introduction

The vibrations absorbers are secondary devices used to minimize the vibrations of the structures. The idea was patented by Frahm in (1911) and has been used in automotive, marine and aerospace applications. An excellent survey of passive, semi-active and active dynamic vibration absorbers was prepared by Sun et al (1995). Young (1952) was the first to consider the application of absorber to control the vibrations of a continuous structure i.e. cantilever beam, at the absorber attachment point with the absorber tuned to the first natural frequency of the beam. Jacquot (1978) used vibration absorbers to eliminate vibration in sinusoidally forced beams by employing a single mode expansion for the beam in an assumed mode approach. Ozguven and Candir (1986) employed a procedure to determine optimum parameters of two viscously damped vibration absorbers to mitigate first two resonances of the beam. Esmailzadeh and Jalili (1998) studied the optimization of the vibration absorber to reduce vibrations of a structurally damped Timoshenko beam, subjected to arbitrary distributed harmonic force excitations. When applying dynamic

*Corresponding e-mail: mailsushil2004@yahoo.co.in
1 Mechanical Department, Sinhgad College of Engineering, Savitribai Phule Pune University, Pune, Maharashtra State, India
vibration absorber to a continuous structure such as beam, vibration can be eliminated only at the attachment point of the beam while amplification of vibration may occur in other parts of the beam. In certain application, it may be of interest to suppress vibration for the particular span of elastic structure where sensitive instruments or components are mounted or attached. Research on suppressing vibration in a region or the particular span of an elastic structure by using the spring mass vibration absorber has been reported recently. Cha (2004,2005) employed spring-mass vibration absorber to reduce vibrations at desired locations by imposing node technique. Cha and Zhou (2006) employed undamped sprung masses and rotational oscillators to create a region of nearly zero amplitudes along the elastic structure for particular driving frequency. Wong et al (2007) developed the combined translational-type absorber and a rotational type absorber for vibration isolation in a region of the beam. Hao and Ripin (2011) reduced handle vibration of grass trimmer by two cantilevered tunable vibration absorbers using the imposing node technique. Patil and Awasare (2016) developed an iterative procedure to find the required resonance frequencies of absorbers to impose node at selected locations on beam.

In the method proposed by Cha to induce multiple nodes, a set of nonlinear algebraic equations need to be solved simultaneously. Numerically, the solution of these equations is very computationally intensive because the convergence is often very slow. The limitations of the procedure developed by Patil and Awasare are that, the maximum allowable absorber amplitudes were not considered while finding the resonance frequencies of the absorbers. The purposes of this paper are:

1) To develop mathematical model of the beam carrying multiple damped absorbers and to formulate the equation for imposing nodes at desired locations on the beam.
2) To develop an algorithm to find the required absorber parameters to impose nodes at selected locations on the beam with constraint of the tolerable vibration amplitudes of the absorber mass.
3) To perform numerical simulation on beam to show the utility of the proposed algorithm.
4) To carry the experimental test to validate the numerical results.

2. Mathematical Model

Fig. 1 shows an arbitrarily supported, Euler-Bernoulli beam with \( n \) tunable vibration absorbers attached at \( x_i \). The absorber modeled as single degree of freedom spring-mass-damper system having mass \( m_i \), stiffness \( k_i \) and damping coefficient \( c_i \) of the \( i \)th absorber. The lumped masses are supported at locations \( x_{mi} \) on the beam. The external harmonic force \( f(t) = Fe^{j\Omega t} \) is applied to the structure at \( x_f \), where \( F \) represents the forcing amplitude, \( \Omega \) denotes the excitation frequency, and \( j = \sqrt{-1} \).
Using the assumed-modes method the deflection of the beam at any point $x$ along the structure is given by (1997)

$$w(x,t) = \sum_{i=1}^{N} \phi_i(x)\eta_i(t),$$  \hspace{1cm} (1)$$
where $N$ is the number of modes used in the assumed-modes expansion, $\phi_i(x)$ are the eigenfunctions of the undamped beam and $\eta_i(t)$ are corresponding generalized co-ordinates. Applying Lagrange’s equations and assuming simple harmonic motion with same response frequency as the excitation frequency, the following equations of motion are obtained

$$
\begin{align*}
\begin{bmatrix} -\omega_x^2 \left[\begin{array}{cc} [M] & 0 \\ 0^T & [m] \end{array}\right] + j\omega_x \left[\begin{array}{cc} [C] & [R_c] \\ [R_c]^T & [c] \end{array}\right] + \left[\begin{array}{cc} [K] & [R_k] \\ [R_k]^T & [k] \end{array}\right] \end{bmatrix} \begin{bmatrix} \eta \\ \zeta \end{bmatrix} &= \begin{bmatrix} F\phi(x_f) \\ 0 \end{bmatrix},
\end{align*}
\end{align*}$$  \hspace{1cm} (2)$$

where $\eta = [\eta_1^T \eta_2^T \cdots \eta_N]^T$, $\zeta = [\zeta_1^T \zeta_2^T \cdots \zeta_n]^T$, and the matrices $[m]$, $[c]$ and $[k]$ of size $n \times n$ are diagonal, whose $i$th elements are given by $m_i$, $c_i$ and $k_i$ respectively. The $N \times N$ $[M]$, $[C]$ and $[K]$ matrices of equation (2) are

$$
[M] = [M^d] + \sum_{i=1}^{L} m_i \phi(x_{m_i})\phi^T(x_{m_i}), \quad [C] = [C^d] + \sum_{i=1}^{n} c_i \phi(x_i)\phi^T(x_i), \quad [K] = [K^d] + \sum_{i=1}^{n} k_i \phi(x_i)\phi^T(x_i),$$  \hspace{1cm} (3)$$

where $[M^d]$, $[C^d]$ and $[K^d]$ are diagonal matrices whose $i$th elements are $M_i$, $C_i$ and $K_i$ are the
generalized masses, damping and stiffnesses of beam. Vector of the eigenfunctions of the beam and the matrices \( [R_c] \) and \( [R_k] \) of size \( N \times n \) are given by

\[
\phi(x_i) = [\phi_1(x_i) \ \phi_2(x_i) \ldots \phi_N(x_i)]^T, \quad \phi(x_f) = [\phi_1(x_f) \ \phi_2(x_f) \ldots \phi_N(x_f)]^T, \\
\phi(x_{mi}) = [\phi_1(x_{mi}) \ \phi_2(x_{mi}) \ldots \phi_N(x_{mi})]^T
\]

\( [R_c] = [-c_1 \phi(x_1) \ldots -c_i \phi(x_i) \ldots -c_n \phi(x_n)] \)

\( [R_k] = [-k_1 \phi(x_1) \ldots -k_i \phi(x_i) \ldots -k_n \phi(x_n)] \) 

(4)

Using second equation of equation (2), the \( \bar{z}_i \) are found to be

\[
\bar{z}_i = -\frac{\omega_i^2 + j \omega_e c_i / m_i}{\omega_i^2 - \omega_e^2 + j \omega_e c_i / m_i} \phi^T(x_i) \eta, \quad i = 1, \ldots, n
\]

In equation (6) resonance frequency \( \omega_i \) and damping coefficient \( c_i \) of \( i \) th absorber are given by

\[
\omega_i = \sqrt{\frac{k_i}{m_i}}, \quad c_i = 2 \zeta_i \omega_i,
\]

(7)

where \( \zeta_i \) is the damping ratio of the \( i \) th absorber

Equation (6) is substituted into the first equation of equation (2) and then solving for \( \eta \) to obtain

\[
\eta = \left\{ -\omega_e^2 [M] + j \omega_e [C^d] + [K^d] + \sum_{i=1}^{S} \sigma_i \phi(x_i) \phi^T(x_i) \right\}^{-1} F \phi(x_f)
\]

(8)

In equation (8)

\[
\sigma_i = \frac{(m_i \omega_e^2 + j c_i \omega_e) \omega_e^2}{\omega_e^2 - \omega_i^2 - j c_i / m_i \omega_e}
\]

(9)

From equation (2) following equation is formulated, the solution of which gives the absorber parameters required to impose node at desired locations \( x_{nr} \) along the beam

\[
W(x_{nr}) = \phi^T(x_{nr}) \left\{ -\omega_e^2 [M] + j \omega_e [C^d] + [K^d] + \sum_{i=1}^{n} \frac{(m_i \omega_e^2 + j c_i \omega_e) \omega_e^2}{\omega_e^2 - \omega_i^2 - j c_i / m_i \omega_e} \phi(x_i) \phi^T(x_i) \right\}^{-1} F \phi(x_f) = 0
\]

(10)

Once the beam with its boundary conditions are specified, absorbers attachment locations \( x_i \), excitation frequency \( \omega_e \) and the excitation location \( x_f \) are known, equation (10) can be used to find the absorber parameters, mass of the absorber \( m_i \) and resonance frequencies \( \omega_i \), for given absorber damping ratio \( \zeta_i \) at which displacements of beam \( W(x_{nr}) \) becomes zero to impose nodes at \( x_{nr} \). An algorithm is developed, which is based on finding the value of resonance frequencies of
absorbers $\omega_i$ at which $W(x_n)$ are minimum so that selected locations $x_m$, becomes nodes along the beam.

The procedure to find the masses $m_1$ and $m_2$ and frequencies of the absorbers $\omega_1$ and $\omega_2$ to impose two nodes

1) Assume $m_1$ and $m_2$. Set initial frequencies of the absorbers $\omega_1 < \omega_e$ and $\omega_2 < \omega_e$
2) Determine $\sigma_1$ and $\sigma_2$ from equation (9)
3) Compute $W(x_{n1})$ using equation (10).
4) Increase the frequency of first absorber $\omega_1$ in steps up to $\omega_1 > \omega_e$ and find $W(x_{n1})$ for each increment
5) Select the frequency $\omega_1$ at which $|W(x_{n1})|$ is minimum
6) Increase the mass $m_1$ of first absorber and repeat steps (2) to (5) till, $|z_1| \leq |z_{1\text{max}}|$. Record the corresponding resonance frequency $\omega_1$
7) Replace frequency and mass of first absorber in $\sigma_1$ with revised frequency $\omega_1$ and mass $m_1$ obtain from step number (6).
8) Compute $W(x_{n2})$ using equation (10)
9) Now increase frequency of second absorber $\omega_2$ in steps up to $\omega_2 > \omega_e$ and find $W(x_{n2})$ for each increment
10) Select the frequency $\omega_2$ at which $|W(x_{n2})|$ is minimum
11) Increase the mass $m_2$ of second absorber and repeat steps (8) to (10) till, $|z_2| \leq |z_{2\text{max}}|$. Record the corresponding resonance frequency $\omega_2$
12) Replace frequency and mass of second absorber in $\sigma_2$ with revised frequency $\omega_2$ and mass $m_2$ obtain from step number (11).
13) Repeat procedure from step number (2) to (12) till the values of $|W(x_{n1})|$ and $|W(x_{n2})|$ converge to zero.

The procedure outlined above will be applied to any arbitrarily supported beam carrying multiple absorbers. It should be noted that if the method does not converge to zero value of $W(x_n)$, the new value of the absorber mass $m_i$ is selected to converge $W(x_n)$ to zero.

3. Numerical Results

Because the assumed-mode method was used to formulate the equations of motions, the proposed procedure can be easily implemented to impose node along any arbitrary supported beam
subjected to harmonic excitations. For cantilever beam, its normalized (with respect to mass per unit length, \( \rho \), of the beam) eigenfunctions \( \phi_i(x) \), generalised masses \( M_i \) and generalised stiffnesses \( K_i \) are given by

\[
\phi_i(x) = \frac{1}{\sqrt{\rho L}} \left( \cos \beta_i x - \cosh \beta_i x + \frac{\sin \beta_i L - \sinh \beta_i L}{\cos \beta_i L + \cosh \beta_i L} \left( \sin \beta_i x - \sinh \beta_i x \right) \right)
\]

(11)

\[
M_i = 1 \quad \text{and} \quad K_i = (\beta_i L)^4 \frac{EI}{(\rho L^4)}
\]

(12)

where \( \beta_i L \) satisfies the following transcendental equation

\[
\cos \beta_i L \cosh \beta_i L = -1
\]

(13)

where \( E \) is Young’s modulus, \( I \) is the moment of inertia of the cross-section of the beam.

In the following example the frequencies and and vibration amplitudes are non-dimensionalised by dividing by \( \sqrt{EI/(\rho L^5)} \) and \( E/(EI/L^3) \) respectively.

Number of modes, \( N = 15 \) is used in the assumed-modes expansion. The value of the absorber frequency \( \omega_i \) and mass \( m_i \) are incremented by \( 0.001 \sqrt{EI/(\rho L^5)} \) and \( 0.0001 \rho L \), in each iteration respectively. The absorber has a low damping to obtain the greatest vibration attenuation at the intended frequency.

Now consider the example of a uniform cantilever beam. It is desired that two nodes to be imposed, at \( x_{n1} = 0.65L \) and \( x_{n2} = 0.75L \), for \( \omega_x = 65 \sqrt{EI/(\rho L^5)} \) at \( x_f = 1L \), for vibration isolation of lumped masses \( m_{l1} = 0.05 \rho L \), \( m_{l2} = 0.1 \rho L \) and \( m_{l3} = 0.05 \rho L \) supported at \( x_{m1} = 0.65L \), \( x_{m2} = 0.7L \) and \( x_{m3} = 0.75L \) respectively. The two absorbers are attached at location \( x_1 = 0.6L \) and \( x_2 = 0.8L \) on the beam. The absorber parameters, resonance frequencies and masses for given damping ratio, required to impose nodes obtained by using the algorithm developed are listed in Table-1.

![Fig. 2. Steady state deformed shapes of beam with and without absorbers tuned to impose nodes at 0.65L and 0.75L.](image-url)
Table 1 Summary of absorber parameters for the example of vibration isolation of lumped masses supported on a uniform cantilever beam

<table>
<thead>
<tr>
<th>Absorber</th>
<th>Damping ratio $\xi_i$</th>
<th>Attachment Locations $x_i$ m</th>
<th>Mass $m_i$ kg</th>
<th>Required Frequency $\omega_i$ rad/sec</th>
<th>Tolerable Mass Amplitude $z_{i,max}$ m</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>0.001</td>
<td>0.6L</td>
<td>0.0685 $\rho L$</td>
<td>$67.89 \sqrt{EI/(\rho L^4)}$</td>
<td>0.02 $F/(EI/L^3)$</td>
</tr>
<tr>
<td>Second</td>
<td>0.001</td>
<td>0.8L</td>
<td>0.0745 $\rho L$</td>
<td>$66.48 \sqrt{EI/(\rho L^4)}$</td>
<td>0.03 $F/(EI/L^3)$</td>
</tr>
</tbody>
</table>

Fig. 2, shows the steady state deformed shapes of cantilever beam with node at 0.65$L$ and 0.75$L$. Note the region between 0.55$L$ and 0.75$L$ experiences less vibration comparing to the beam without absorbers.

4. Experimentation

For imposing two nodes the dual cantilevered mass absorbers were designed and constructed as shown in Fig. 3. The resonance frequency of device is adjusted by moving the masses towards or away from the base support, which alters the effective stiffness in the system and alters its resonance frequency. Note that the structural damping of the absorbers used is considered as equivalent viscous damping, see Thomson & Dahleh (1997) and Dayou & Brennan (2002). The experimental modal analysis of the absorber was carried out to determine the resonance frequencies of absorber for different mass positions on the threaded rods. Fig. 4 shows the experimental modal analysis setup for absorber. The impact hammer (Kistler 9722A2000) was used to excite the absorber mass, while light-weight ICP accelerometer (PCB 333B32) fixed to mass was used to measure the frequency response read from a FFT vibration analyzer (Adash 4300). Fig. 5 describe the relationship between the mass position and the resonance frequencies of the absorbers.

Fig. 3. Variable stiffness dual mass vibration absorbers used for experimental test
Fig. 4. Experimental modal analysis of absorber

Fig. 5. Variation of resonance frequencies of a) absorbers attached at 0.6L b) absorber attached at 0.8L, with mass position.
Table 2 The system parameters and material properties used in the experimental test.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the beam $L$</td>
<td>1 m</td>
</tr>
<tr>
<td>Thickness of the beam $t$</td>
<td>0.01 m</td>
</tr>
<tr>
<td>Width of the beam $b$</td>
<td>0.065 m</td>
</tr>
<tr>
<td>Density of the beam</td>
<td>7830 Kg/m³</td>
</tr>
<tr>
<td>Young's modulus of the beam $E$</td>
<td>$2.1 \times 10^{11}$ N/m²</td>
</tr>
<tr>
<td>Mass per unit length of beam $\rho$</td>
<td>5 Kg/m</td>
</tr>
<tr>
<td>Parameter $E I / L^3$ used to nondimensionalised stiffness of beam</td>
<td>1137.5 N/m</td>
</tr>
<tr>
<td>Parameter $\sqrt{E I / (\rho L^2)}$, used to nondimensionalised frequencies</td>
<td>14.95 N/m/Kg</td>
</tr>
<tr>
<td>Parameter $F / E I / L^3$ used to nondimensionalised vibration amplitude of beam and absorber masses</td>
<td>0.004 m</td>
</tr>
<tr>
<td>Weight of lumped masses</td>
<td></td>
</tr>
<tr>
<td>$m_l_1$</td>
<td>0.25 Kg = 0.05 $\rho L$</td>
</tr>
<tr>
<td>$m_l_2$</td>
<td>0.5 Kg = 0.1 $\rho L$</td>
</tr>
<tr>
<td>$m_l_3$</td>
<td>0.25 Kg = 0.05 $\rho L$</td>
</tr>
<tr>
<td>Total weight of end masses of first absorber $m_1$</td>
<td>0.342 Kg = 0.0685 $\rho L$</td>
</tr>
<tr>
<td>Total weight of end masses of second absorber $m_2$</td>
<td>0.372 Kg = 0.0745 $\rho L$</td>
</tr>
<tr>
<td>Damping ratio of absorbers</td>
<td>$\xi_1 = \xi_2$ 0.001</td>
</tr>
</tbody>
</table>
Next in order to verify the numerical result, experimental test was conducted for the case of cantilever beam with two absorbers as shown in Fig. 6. The two absorbers are attached at $0.6L$ and $0.8L$ and the harmonic input force is applied at the tip of the beam. The force amplitude is kept constant at 5 N through the experiment and used to non-dimensionalize the displacements of the beam by dividing by $F/(EI/L^3)$. The system parameters and material properties used in experimental test are listed in Table 1. The absorbers were tuned by moving the end masses in or out such that the displacement at $0.65L$ and $0.75L$ was minimized. After tuning of the absorbers the vibration amplitudes were measured at twenty points on the beam’s surface by the accelerometer and recorded by vibration analyzer to plot experimental steady state response as shown in Fig. 7. It is observed that the vibrations at the node locations of the beam are reduced to a minimum level. Comparing Fig. 7, with Fig. 2, it is observed that there is good agreement between the numerical results and experimental results.

![Graph](image)

**Fig.7.** Experimental steady state response of the beam with and without absorbers tuned to impose nodes at $0.65L$ and $0.75L$.

5. Conclusions

In this paper, use of tunable vibration absorbers has been investigated to reduce vibration of the segment of beam by imposing nodes method. The algorithm given to find the absorber parameters is efficient, simple to code and can be used for beam with different boundary conditions. The design constraint on maximum allowable vibration amplitudes on absorber masses makes the algorithm developed more practical. Numerical simulation shows that the vibrations are suppressed to the greater extent for the desired segment of beam. Experimental tests were performed to validate procedure given, and good agreements were found between numerical solutions obtained by proposed scheme and experimental results.

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